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BAIRE SPACES AND HYPERSPACES

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This paper examines the question as to when the hyperspace of a Baire space is a Baire space, and related questions. An answer is given in terms of a certain product space's being a Baire space.

A hyperspace of a space X is the space of closed subsets of X under a natural topology. In this paper we investigate what happens to Baire spaces in the formation of hyperspaces. The first section is devoted to a discussion of the basic concepts. In the second section some characterizations of Baire spaces are given which will be useful while working with hyperspaces, the study of which occurs in the third section. In particular, we shall be primarily concerned with two basic questions. If X is a Baire space, when is the hyperspace of X a Baire space? If the hyperspace of X is a Baire space, when is X a Baire space? We also look briefly at Baire spaces in the strong sense and pseudo-complete spaces.

1. Basic definitions and properties. A *Baire space* is a space in which every countable intersection of dense open subsets is dense. It can also be defined as a space such that every nonempty open subspace is of second category. The usual definition of a space of *first category* is one which can be written as a countable union of nowhere dense subsets (i.e., subsets whose closures have no interior points). A space is of *second category* then if it is not of first category. Also second category spaces can be characterized as spaces in which every countable intersection of dense open subsets is nonempty.

We now list a few of the properties which the Baire space concept enjoys. Of course the Baire Category Theorem gives a sufficient condition for a space to be a Baire space. That is, every complete metric space is a Baire space. Also every locally compact Hausdorff space is a Baire space. Clearly every nonempty open subspace of a Baire space is a Baire space. In fact a space is a Baire space if and only if every point has a neighborhood which is a Baire space. A useful property of Baire spaces is that every space which contains a dense Baire subspace is a Baire space.

The question as to which products of Baire spaces are Baire spaces has been a difficult one. There have been very few different examples