PACIFIC JOURNAL OF MATHEMATICS
Vol. 59, No. 2, 1975

ON SEMI-SIMPLE GROUP ALGEBRAS (I)

EUGENE SPIEGEL AND ALLAN TROJAN

For F a field and G a group, let FG denote the group algebra of G over F. Let $\mathscr{G}$ be a class of finite groups, and $\mathscr{F}$ a class of fields. Call the fields F_1 and F_2 ($F_i \in \mathscr{F} i = 1, 2$) equivalent on $\mathscr{G}$ if for all $G, H \in \mathscr{G}$, $F_1G \simeq F_1H$ if and only if $F_2G \simeq F_2H$. In this note we begin a study of this equivalence relation, taking the case where $\mathscr{G}$ consists of all finite p-groups and $\mathscr{F}$ those fields F, for which FG is simi-simple for all $G \in \mathscr{G}$.

1. Let ζ_n denote a primitive n^{th} root of unity (over the field under consideration). Throughout the paper, p will denote a fixed odd prime, and all fields will be assumed to be of characteristic distinct from p.

For reference, we begin with a result from field theory.

PROPOSITION 1.1. *Suppose F is a field and Ω an extension field of F. Let $K_i = 1, 2$ be fields such that $F \subset K_i \subset \Omega$, and assume that K_1/F is a finite Galois extension. Then the following are equivalent.*

(i) *K_1 and K_2 are linearly disjoint over F*

(ii) *$K_1 \otimes_F K_2$ is a field*

(iii) *$K_1 \otimes_F K_2 \simeq K_1K_2$*

(iv) *$K_1 \cap K_2 = F$.*

Proof. See [1] page 78 and page 149.

Let G be a group of order p^n and K a field. We discuss the structure of KG.

By Maschke's theorem $KG \simeq \sum A_i$, where $A_i \simeq [K]_{n_i} \otimes D_i$, D_i is a finite dimensional division algebra over K, and $[K]_{n_i}$ denotes the ring of $n_i \times n_i$ matrices over K.

If K is a perfect field, then $A_1 \simeq K$ and for $i \neq 1$, the center of D_i is isomorphic to $K(\chi_i) = K(\{\chi_i(g) \mid g \in G\})$, for some nonprincipal irreducible character χ_i of G, $K(\chi_i) \subset K(\zeta_{p^n})$. If T is any nonprincipal irreducible representation of G into the $m \times m$ matrices over an algebraically closed field F then $T(G)$ is a finite p-group and thus contains an element S of order p in its center. Since T is irreducible, this central element must be a scalar matrix of order p, that is, a scalar matrix with diagonal element ζp.

Let χ_T be the character associated with T. Then there is an