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# METRIZATION OF SPACES WITH COUNTABLE LARGE BASIS DIMENSION

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**With the following results, we generalize known metrization theorems for spaces with large basis dimension 0 i.e., non-archimedean spaces) to the higher dimensions:** *Theorem.* **If $X$ is a normal $\Sigma$-space with countable large basis dimension, then $X$ is metrizable.** *Theorem.* **If $X$ is a normal $w\Delta$-space with countable large basis dimension, then $X$ is metrizable.**

**I. Introduction.** A collection $\Gamma$ of subsets of a set $X$ is said to have *rank* 1 if whenever $g_1$ and $g_2$ are in $\Gamma$ with $g_1 \cap g_2 \neq \varnothing$ then $g_1 \subset g_2$ or $g_2 \subset g_1$. According to P. J. Nyikos [13], a topological space $X$ has *large basis dimension* $\leqq n$ (denoted $\operatorname{Bad} X \leqq n$) if $X$ has a basis which is the union of $n+1$ rank 1 collections of open sets. $X$ has *countable large basis dimension* ($\operatorname{Bad} X \leqq \aleph_0$) if $X$ has a basis which is the union of a countable number of rank 1 collections such that each point of $X$ has a basis belonging to one of the collections (a property which is automatically true in the finite case). $\operatorname{Bad} X$ coincides with $\operatorname{Ind} X$ and $\dim X$ for metric spaces.

Spaces having large basis dimension 0 are usually called *non-archimedean* spaces. Theorems of Nyikos [11] and A. V. Archangelskii [3] show that a non-archimedean space is metrizable if and only if it is a $\Sigma$-space or a $w\Delta$-space. In this paper we show that these results are valid, under mild assumptions, for the higher dimensions. Our results also improve a result of G. Gruenhage [6], who showed that compact spaces having finite large basis dimension are metrizable.

**II. Main results.** According to Nyikos [11], a *tree of open sets* is a collection $\Gamma$ of open sets such that if $g \in \Gamma$, then the set $\{g' \in \Gamma \mid g' \supset g\}$ is well-ordered by reverse inclusion; that is, $g \leqq g'$ if and only if $g \supset g'$. Nyikos shows that the rank 1 collections for spaces with $\operatorname{Bad} X \leqq \aleph_0$ can be considered as rank 1 trees of open sets. The following fact will be used in our proofs:

LEMMA 1. *Let $T$ be a rank 1 tree of open subsets of a regular space $X$ which contains a basis at each point of a subset $X'$ of $X$. Then if $\mathscr{U}$ is a cover of $X'$ by open subsets of $X$, there exists a subset $T'$ of $T$ such that*

(i) *$T'$ is a cover of $X'$;*
(ii) *the elements of $T'$ are pairwise disjoint;*