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SPINOR NORMS OF LOCAL INTEGRAL ROTATIONS, II

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Dedicated to Professor Olga Taussky Todd's Birthday

The spinor norms of integral rotations on an arbitrary quadratic form over a dyadic local field in which 2 is prime are determined. Results are stated in terms of the components of a Jordan splitting of the given form. Results obtained are applied to improve a theorem of Kneser giving sufficient conditions for an indefinite Z-lattice to have class number 1.

The behavior of integral quadratic forms over a global field can be partially described in terms of the local behavior relative to each of the prime spots on the field. In particular, in computing the number of spinor genera in the genus of a given form, it is necessary to compute the spinor norm of the group of local integral rotations at each prime spot (see e.g. [3], [4]). These computations have been performed whenever the local form is modular (see [2]). In the case of an arbitrary form, the Jordan splitting can be used to decompose the given form as an orthogonal sum of modular forms. In the present article we deal with the problem of obtaining the desired spinor norm by using these modular components. When the spot in question is nondyadic, this problem has been solved by Kneser in [3]. We handle the case of a dyadic spot in which 2 is prime. The significance of the restriction of 2 being prime is that strong use is made of theorems on the generation of the local integral orthogonal groups in this case (see [5]) which are not known for arbitrary dyadic local fields.

We adopt the notation of [4]. So we will consider a lattice L over a dyadic local field F in which 2 is prime. Denote the integers and units in F by $\mathfrak{o}$ and $\mathfrak{U}$, respectively; $\Delta = 1 + 4\rho$ denotes a nonsquare unit of quadratic defect $4\mathfrak{o}$. To emphasize the distinction between spaces over F and lattices over $\mathfrak{o}$, we will use $[\alpha_1, \cdots, \alpha_n]$ to denote spaces and $\langle\alpha_1, \cdots, \alpha_n\rangle$ to denote lattices. $(,)$ will denote the Hilbert symbol on F and θ the spinor norm function.

For any lattice L, the Jordan decomposition of L can be obtained as in [4]. We determine $\theta(0^+(L))$, where $0^+(L)$ denotes the group of rotations of L, in terms of invariants of the Jordan components. The paper is divided into 4 sections. In the first we perform the calculations for the binary case. The second and third deal with the various possible types of Jordan decompositions and the fourth section shows an application of these calculations to improve the