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STOCHASTIC CONVEX PROGRAMMING: SINGULAR MULTIPLIERS AND EXTENDED DUALITY SINGULAR MULTIPLIERS AND DUALITY

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A two-stage stochastic programming problem with recourse is studied here in terms of an extended Lagrangian function which allows certain multipliers to be elements of a dual space $(\mathscr{L}^\infty)^*$, rather than an $\mathscr{L}^1$ space. Such multipliers can be decomposed into an $\mathscr{L}^1$-component and a "singular" component. The generalization makes it possible to characterize solutions to the problem in terms of a saddle-point, if the problem is strictly feasible. The Kuhn-Tucker conditions for the basic duality framework are modified to admit singular multipliers. It is shown that the optimal multiplier vectors in the extended dual problem are, in at least one broad case, ideal limits of maximizing sequences of multiplier vectors in the basic dual problem.

1. Introduction. This paper is a sequel to **[3]**, where the following two-stage stochastic programming problem was investigated: minimize

$$f_{10}(x_1)+\int_S f_{20}(s, x_1, x_2(s))\sigma(ds) \tag{1.1}$$

over all $x_1 \in R^{n_1}$ and $x_2 \in \mathscr{L}^\infty_{n2} = \mathscr{L}^\infty(S, \Sigma, \sigma; R^{n_2})$ satisfying

$$x_1 \in C_1 \quad \text{and} \quad f_{1i}(x_1) \leqq 0 \quad \text{for} \quad i = 1, \cdots, m_1 \tag{1.2}$$

and almost surely

$$x_2(s) \in C_2 \quad \text{and} \quad f_{2i}(s, x_1, x_2(s)) \leqq 0 \quad \text{for} \quad i = 1, \cdots, m_2. \tag{1.3}$$

Here (S, Σ, σ) denotes a probability space, and the sets $C_1 \subset R^{n_1}$ and $C_2 \subset R^{n_2}$ are closed, convex, and nonempty. The functions f_{1i} for $i = 0, 1, \cdots, m_1$ and $f_{2i}(s, \cdot, \cdot)$ for $i = 0, 1, \cdots, m_2$ are finite and convex on all of R^{n_1} and $R^{n_1} \times R^{n_2}$, respectively. It is assumed for each $(x_1, x_2) \in R^{n_1} \times R^{n_2}$ the function $f_{2i}(\cdot, x_1, x_2)$ is measurable on S, in fact summable if $i = 0$ and bounded if $i = 1, \cdots, m_2$. (As pointed out in **[1]**, these assumptions imply that for every $x_1 \in R^{n_1}$ and $x_2 \in \mathscr{L}^\infty_{n2}$, $f_{2i}(s, x_1, x_2(s))$ is measurable in s, summable if $i = 0$ and essentially bounded if $i = 1, \cdots, m_2$).