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# ON FINITE HANKEL TRANSFORMATION OF GENERALIZED FUNCTIONS

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**In this paper the finite Hankel transformation of generalized function of a certain space is defined, and an inversion formula for the transformation is established. The inversion formula gives rise to a Fourier–Bessel series expansion of generalized functions. The convergence of the series is interpreted in the weak distributional sense. An operation transform formula is also obtained, which together with the inversion formula, is applied in solving certain distributional differential equations.**

**1. Introduction.** The orthonormal series expansions of distributions of certain classes have been studied by Gelfand and Shilov [**3**, Vol. 3], Giertz [**2**], Walter [**9**] and Zemanian [**11**]. Some previous works on Fourier and Hermite series expansions of certain classes of distributions are due to Schwartz [**8**, Vol. II] and Korevaar [**5**]. The procedures of Giertz, Walter, and Korevaar are appropriate for the fundamental sequence approach to generalized functions (see Lighthill [**6**] and Korevaar [**4**]), whereas those of Schwartz, Gelfand and Shilov and Zemanian, including the present work are suitable to functional approach to generalized functions.

Some particular cases of orthonormal series expansions including Fourier–Bessel series expansion of generalized functions have also been studied by Zemanian [**11**]. The method involved in his work is very much related to Hilbert space techniques. The present goal is to extend the classical inversion theorem for finite Hankel transform [**10**, p. 591] to a class of generalized functions, which gives rise to the Fourier–Bessel series expansion of the generalized function; the convergence of the series is interpreted in the weak distributional sense. The techniques developed in establishing the present inversion theorem are quite different from those employed in previous works and, at the same time, are quite simple and handy. An operation transform formula is also established and is applied in solving certain distributional differential equations.

The finite Hankel transformation of a function $f(t)$ defined on the interval $(0, 1)$ is defined as

$$H(m) = \int_0^1 tf(t)J_\nu(j_m t)dt, \qquad m = 1, 2, 3, \cdots, \tag{1.1}$$