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A SHAPE THEORY WITH SINGULAR HOMOLOGY

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A modified concept of a shape of a topological space is introduced which allows some basic geometric constructions: (1) One has a convenient homotopy concept which originates from a cylinder functor. (2) All inclusions of compact metric spaces are cofibrations. (3) Shape mappings which agree on the intersection of their counterimages can be pasted together (existence of push-outs). (4) There exists a singular complex $\bar{S}$ which has the same properties for shape mappings as the ordinary singular complex S for continuous maps. (5) Consequently one has a singular (shape) homology which for compact metric spaces turns out to be isomorphic to the (shape-theoretically defined) homotopical homology (in the sense of G. W. Whitehead) and to the Steenrod-Sitnikov homology.

0. Introduction. Two topological spaces X and Y are supposed to be of the same shape whenever they have similar geometric properties. In order to give this concept a precise meaning one has to define an appropriate shape category S with topological spaces as objects but with a new class of morphisms (other than continuous mappings or their homotopy classes) the so called shape mappings. This category should permit most of the geometric constructions which constitute the value of the category of CW-spaces, to be performed.

The first model of a shape category was introduced 1968 by K. Borsuk. Four years later S. Mardešić [7] gave a rather simple characterization of Borsuk's shape category S. Roughly speaking S turned out to be the universal category (with topological spaces as objects) in which two spaces X, Y are equivalent whenever they cannot be distinguished by those homotopy invariants which are determined by mappings of X resp. Y into arbitrary CW-spaces. Thus Čech cohomology is (but singular homology is not) a shape invariant.

In the present paper we present a modified shape category $\bar{\boldsymbol{K}}$ which has the following property:

Let $X = X_1 \cup X_2$, $D = X_1 \cap X_2$ and $f_i \in \bar{\boldsymbol{K}}(X_i, Y)$, $i = 1,2$ be morphisms which coincide on D, then there exists a unique $f \in \bar{\boldsymbol{K}}(X, Y)$ which restricts to f_i on X_i (Lemma 3.1, 3.2, Proposition 3.3). In other words: Mappings can be pasted together.

Although $\bar{\boldsymbol{K}}$ is not a homotopy category, it does have a natural notion of homotopy so one can define fibrations and cofibrations within