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THE DUNFORD-PETTIS PROPERTY FOR CERTAIN UNIFORM ALGEBRAS

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A Banach space B has the Dunford-Pettis property if $x_n^*(x_n) \to 0$ whenever $x_n \to 0$ weakly and the sequence x_n^* tends to zero weakly in B^* (i.e. $\sigma(B^*, B^{**})$). Suppose now that A is a uniform algebra on a compact space X. If ϕ is a nonzero multiplicative linear functional on A then M_ϕ is the set of positive representing measures of ϕ. If A is such that a singular measure which is orthogonal to A must necessarily be zero and if all M_ϕ are weakly compact sets then the algebra A as well as its dual have the Dunford-Pettis property.

The idea of the proof is that A^* the dual of A can be decomposed into components for which the results of Chaumat [1] and Cnop-Delbaen [2] can be applied. The fact that an l_1 sum of Dunford-Pettis spaces is also a Dunford-Pettis space then gives the result. In paragraph two some conditions ensuring the weak compactness of M_ϕ are given. These conditions are related to those used in the definition of core and enveloping measures (see [6]).

1. Notation and preliminaries. X will be a compact space, $A \subset \mathscr{C}(X)$ a closed subalgebra of the space of continuous complex-valued functions on X. The algebra A is supposed to contain the constants and to separate the points of X. The spectrum M_A is the set of all nonzero multiplicative linear functionals on A. If $\phi \in M_A$ then M_ϕ is the set of all positive measures on X representing ϕ, i.e.

$$M_\phi = \left\{\mu \in M(X) \mid \mu \geqq 0 \text{ and } \forall f \in A \text{ we have } \phi(f) = \int f d\mu\right\} .$$

As well known M_ϕ is a convex set, compact for the topology $\sigma(M(X), \mathscr{C}(X))$. We say that two multiplicative linear forms ϕ and ψ belong to the same Gleason part if $\|\phi - \psi\| < 2$ in A^*, the dual of A. It is well known that being in the same Gleason part is an equivalence relation and hence $M_A = \bigcup_{\pi \in \Pi} \pi$ where Π is the set of all Gleason equivalence classes. For more details and any unexplained notion on uniform algebras we refer to [6].

If E is a Banach space then E has the Dunford-Pettis property if $e_n^*(e_n) \to 0$ whenever $e_n \to 0$ weakly and $e_n^* \to 0$ weakly (i.e. $\sigma(E^*, E^{**})$).

For more details and properties of such spaces see Grothendieck