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THE CENTRALISER OF $E \otimes_\lambda F$

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If E is a real Banach space then $\mathscr{B}(E)$ is the space of all bounded linear operators on E, and $\mathscr{Z}(E)$ the subspace of M-bounded operators, i.e. the centraliser of E. Two Banach spaces E and F are considered as well as the tensor product $E \otimes_\lambda F$. There is a natural mapping of the algebraic tensor product $\mathscr{Z}(E) \odot \mathscr{Z}(F)$ into $\mathscr{Z}(E \otimes_\lambda F)$. It is shown that $\mathscr{Z}(E \otimes_\lambda F)$ is precisely the strong operator closure, in $\mathscr{B}(E \otimes_\lambda F)$, of its image.

1. Definitions and statement of results. A linear operator T on a real Banach space E is *M-bounded* if there is $\lambda > 0$ such that if $e \in E$ and D is a closed ball in E containing λe and $-\lambda e$, then $Te \in D$. The *centraliser* of E, $\mathscr{Z}(E)$, is the commutative Banach algebra of all M-bounded linear operators on E. Let K denote the unit ball of E^*, the Banach dual of E, equipped with the weak* topology. We denote the set of extreme points of a convex set C by $\mathscr{E}(C)$. In [2], Theorem 4.8 it is shown that a bounded linear operator T on E is M-bounded if and only if each point of $\mathscr{E}(K)$ is an eigenvalue for T^*, the adjoint of T. Thus there is a real valued function $\tilde{T}$ on $\mathscr{E}(K)$ such that $T^*p = \tilde{T}(p)p (p \in \mathscr{E}(K))$.

An *L-ideal* in a real Banach space is a subspace I with a complementary direct summand J such that $\|i\| + \|j\| = \|i + j\| (i \in I, j \in J)$. The sets $I \cap \mathscr{E}(K)$ for I a weak*-closed L-ideal in E^* form the closed sets of the structure topology on $\mathscr{E}(K)$. The map $T \mapsto \tilde{T}$ is an isometric algebra isomorphism of $\mathscr{Z}(E)$ onto the bounded structurally continuous real valued functions on $\mathscr{E}(K)$ with the supremum norm and pointwise multiplication ([2], Theorem 4.9).

We shall consider two Banach spaces E and F, K will retain its meaning and M will denote the corresponding subset of F^*. We use $E \odot F$ to denote the algebraic tensor product of E and F. We shall consider the norm

$$\left\| \sum_{i=1}^{n} e_i \otimes f_i \right\|_\lambda = \sup \left\{ \left| \sum_{i=1}^{n} k(e_i) m(f_i) \right| : k \in K,\ m \in M \right\} .$$

$E \odot_\lambda F$ will denote $E \odot F$ with this norm, and $E \otimes_\lambda F$ its completion.

We may identify $E \otimes_\lambda F$ concretely in a number of ways. The formula $(k, m) \mapsto \sum_{i=1}^{n} k(e_i) m(f_i)$ defines a real valued function on $K \times M$. Such functions are continuous and affine in each variable. $\| \sum_{i=1}^{n} e_i \otimes f_i \|_\lambda$ is the same as the supermum norm for such a function, so we may identify $E \otimes_\lambda F$ with a subspace H, the closure of