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ON THE DIFFERENTIABILITY OF MULTIFUNCTIONS

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A new concept of differential for a multifunction is introduced. Here by a multifunction we mean a map from a Banach space X to some specified family of non void subsets of a Banach space Y. The comparison with another definition due to Lasota and Strauss shows that if a multifunction admits both differentials, these must be equal. The results are applicable to the perturbation theory for multivalued differential equations in a Banach space

$$\dot{x} \in F(x)$$

in a neighborhood of a rest position.

1. Introduction. The concept of differentiability for multifunctions has been considered by many authors from different points of view (**[1]**, **[3]**, **[8]**, **[11]**, **[13]**, **[17]**, **[18]**). Of all these approaches, that developed by Lasota and Strauss **[17]** seems to be more useful in perturbation theory for ordinary differential equations in the real Euclidean space R^n. Further applications along this same direction were obtained in **[10]** (see also **[9]**). In the present paper, moving from an idea of Bridgland **[3]**, a new concept of differentiability for a multifunction is studied. This notion seems to be useful in perturbation theory. In **[7]** an application to problems of stability for multivalued differential equations in Banach spaces is given.

The definitions and the main properties of a multivalued differential (i.e. the differential of a multifunction or, in particular, of a function) are contained in §§2 and 3. Now, it is perhaps better to start by giving an answer to the preliminary question: where one may encounter a multivalued differential. To this end we recall the well known Theorems 1.1 and 1.2, due to Lyapunov.

THEOREM 1.1 (**[20]** p. 222). *Let f: $R^n \to R^n$, $f(0)=0$, be continuously differentiable in a neighborhood of the origin with Fréchet differential f' at the origin. Let all the eigenvalues of f' have negative real parts, i.e. the origin is asymptotically stable for*

$$(1.1) \qquad \dot{x} = f'(x).$$

Then the origin is asymptotically stable for

$$(1.2) \qquad \dot{x} = f(x).$$