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TOPOLOGIES ON THE SET OF CLOSED SUBSETS

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In this paper the techniques of Nonstandard Analysis are used to study topologies on the set $\Gamma(X)$ of closed subsets of a topological space X. The first section of the paper investigates the "compact" topology developed by Narens and constructs a variant of that topology which is particularly useful for non locally compact spaces X. (When X is locally compact this variant is shown to be identical with Naren's original "compact" topology.) This new topology is a natural extension to $\Gamma(X)$ of the one point compactification of X embedded in $\Gamma(X)$ in the obvious way with the point at infinity corresponding to the empty set. The second section shows that the techniques developed by Narens can be used to obtain a natural characterization of the Vietoris Topology by considering monads of non nearstandard points. The final section uses this same approach to construct a topological analog of the Hausdorff metric for normal spaces.

0. Introduction. Suppose that X is a topological space and that Γ denotes the set of closed subsets of X. It is frequently desirable to endow Γ with a topology of its own. Various topologies on Γ have been proposed and studied by several mathematicians. If X is a metric space, Hausdorff (see **[2]**, **[6]**, **[7]**) defined a metric on Γ in a natural way. With this metric X is embedded isometrically as a closed subset of Γ by the mapping $x \mapsto \{x\}$. One drawback of this metric, however, is that it depends in an essential way on the metric on X. That is, d and d' may be two metrics for the same topology on X, but induce Hausdorff metrics which do not give the same topology on Γ. In **[10]** E. Michaels investigates among other topologies the Vietoris or Finite topology on Γ. This topology also has the property that X is embedded as a closed subset of Γ by the mapping $x \mapsto \{x\}$. Both the Hausdorff metric on Γ and the Vietoris topology on Γ make Γ into a compact space if and only if X was originally compact. More recently, L. Narens **[12]** has introduced an interesting topology on Γ using the techniques of Nonstandard Analysis. This topology always makes Γ a compact set with the empty set $\varnothing \in \Gamma$ acting (see Theorem I.8) somewhat like the point at infinity of the one-point compactification of X.

Nonstandard Analysis provides a particularly nice framework for investigating topological questions. Intuitively, a topological space is a set together with some notion of "nearness"