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TOPOLOGICAL GROUPS WHICH SATISFY AN OPEN MAPPING THEOREM

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Let $\mathscr{C}$ be a category of Hausdorff topological groups. A Hausdorff topological group G is called a $B(\mathscr{C})$ group if every continuous and almost open homomorphism from G onto a group in $\mathscr{C}$ is open. An internal characterization of such groups is obtained. For certain $\mathscr{C}$, the permanence properties of $B(\mathscr{C})$ groups and related categories are investigated, with some positive results pertaining to products and subobjects, and several counterexamples. Forms of the closed graph theorem for topological groups are then obtained which generalize results of T. Husain.

1. Definitions and permanence properties. Given a topological group G with topology u, we shall denote the filter of neighbourhoods of the identity by $\mathscr{V}(G)$ or $\mathscr{V}(u)$, and closures by Cl_G or Cl_u, depending on the emphasis desired. If u and v are two group topologies on a group G, then $v(u)$ will denote that group topology on G having as a fundamental system of unit neighbourhoods the collection $\{\mathrm{Cl}_v U\colon U \in \mathscr{V}(u)\}$. The set of closed normal subgroups of a topological group G will be denoted by $\mathscr{N}(G)$. A homomorphism $f\colon G \to H$ of topological groups is said to be almost open (resp., almost continuous) if the image (resp., inverse image) of a unit neighbourhood is dense in a unit neighbourhood. An isomorphism of topological groups is a group isomorphism which is both continuous and open.

Let $\mathscr{C}$ be a category of Hausdorff topological groups. After **[8]**, we say that a Hausdorff group G is a $B(\mathscr{C})$ group if every continuous and almost open homomorphism from G onto a group in $\mathscr{C}$ is open, and that G is a $B_r(\mathscr{C})$ group if every homomorphism with these properties which is also one-to-one is open. We reserve the symbol $\mathscr{A}$ for the category of all Hausdorff topological groups.

Husain **[8]** showed that locally compact groups and complete metrizable groups are $B(\mathscr{A})$ groups, while Brown [**2**, Theorem 4] showed that any topological group complete in the sense of Čech has the $B(\mathscr{A})$ property. A minimal topological group (i.e., one with its coarsest compatible Hausdorff topology) is easily seen to be a $B_r(\mathscr{A})$ group. Other examples will be mentioned later.

Husain also observed [**8**, Theorem 31.4] that a topological group (G, u) is a $B_r(\mathscr{A})$ group iff, for every Hausdorff group topology v on G such that $v \subseteq u$ and $v(u) = v$, it follows that $u = v$. We give analogous