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THE RANGE OF ANALYTIC EXTENSIONS

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Denote by Δ, $\bar{\Delta}$, $\partial\Delta$ the open unit disc in C, its closure and its boundary, respectively. Let X be a complex Banach space and denote by $\mathscr{A}(X)$ the class of all non-empty sets $P \subset X$ having the following property: given any closed set $F \subset \partial\Delta$ of measure 0 and any continuous function $f: F \to P$ there exists a continuous extension $\tilde{f}: \bar{\Delta} \to X$ of f, analytic on Δ and satisfying $\tilde{f}(\bar{\Delta} - F) \subset \operatorname{Int} P$.

THEOREM. **$P \in \mathscr{A}(X)$ if and only if $\operatorname{Int} P$ is connected, locally connected at every point of P and satisfies $P \subset$ closure $(\operatorname{Int} P)$.**

THEOREM. **If $P \subset C$ consists of more than one point then $P \in \mathscr{A}(C)$ if and only if given any F and f as above there exists a continuous extension $\hat{f}: \bar{\Delta} \to C$ of f, analytic on Δ and satisfying $\hat{f}(\bar{\Delta}) \subset P$.**

This generalizes a theorem of Rudin which asserts that such $\hat{f}$ exists if $P \subset C$ is homeomorphic to $\bar{\Delta}$.

THEOREM. **If $P \in \mathscr{A}(X)$ then given any relatively open set $B \subset \partial\Delta$, any relatively closed set $F \subset B$ of measure 0 and any continuous function $f: F \to P$ there exists a continuous extension $\tilde{f}: \Delta \cup B \to X$ of f, analytic on Δ and satisfying $\tilde{f}((\Delta \cup B) - F) \subset \operatorname{Int} P$.**

0. Introduction. Throughout, we denote by Δ, $\bar{\Delta}$ and $\partial\Delta$ the open unit disc in C, its closure and its boundary, respectively. If X is a complex Banach space and $r > 0$ we write $B_r(X) = \{x \in X: \|x\| < r\}$. Let $x \in X$ and $S, T \subset X$. We write $x + S = \{x + u: u \in S\}$ and $S + T = \{u + v: u \in S, v \in T\}$. We denote by $\operatorname{Int} S$, $\bar{S}$ the interior of S and the closure of S, respectively. If F is a compact Hausdorff space we denote by $C(F, X)$ the set of all continuous functions from F to X and write $C(F)$ for $C(F, C)$. If $B \subset \partial\Delta$ is a relatively open set we denote by $H_B(\Delta, X)$ the set of all continuous functions from $\Delta \cup B$ to X which are analytic on Δ. For $H_{\partial\Delta}(\Delta, X)$ we write $A(\Delta, X)$ and for $A(\Delta, C)$, the disc algebra, we write $A(\Delta)$. We denote the set of all positive integers by N. If $a, b \in R$, $a < b$ we write $[a, b] = \{t \in R: a \leqq t \leqq b\}$ and we denote $[O, 1]$ by I.

The well known Rudin-Carleson theorem [3, 19, 22] states that given a closed set $F \subset \partial\Delta$ of measure 0 and $f \in C(F)$ there exists an extension $\tilde{f} \in A(\Delta)$ of f satisfying

$$\max_{z \in \bar{\Delta}} |\tilde{f}(z)| = \max_{s \in F} |f(s)| .$$