PACIFIC JOURNAL OF MATHEMATICS
Vol. 70, No. 1, 1977

THE PRESERVERS OF ANY ORTHOGONAL GROUP

E. P. BOTTA AND STEPHEN PIERCE

Let L be an invertible linear map on the space $M(n, k)$ of n-square matrices over a field k of characteristic not 2. In this paper we classify all such L which preserve a particular orthogonal group of a nonsingular symmetric bilinear form. We use some elementary facts about algebraic groups and an idea of Dieudonné's. There is some indication that our use of an algebraic geometric setting is the proper one for many problems of this type.

There are a considerable number of results which may be paraphrased as follows: "let $L: M(n, k) \to M(n, k)$ be a linear transformation that preserves some property related to matrix multiplication. Then L is almost an inner automorphism of $M(n, k)$." While the statement of these results exhibits a large degree of similarity, an examination of the proofs reveals almost no similarity. The property in question could be determinant, nonsingularity, or orthogonality.

For examples of such results, see [**4, 5, 6, 7, 8, 10**]. In particular, an excellent survey is in [**6**].

We remark that we must assume L is invertible, since Wei's results [**10**] show that if singular maps are allowed, then pathological cases can occur.

1. Notation.

Let K be an algebraically closed field of characteristic not 2. If n is a positive integer, K^n will be the vector space of n-tuples of elements of K, $M(n, k)$ the algebra of n-square matrices over K, $\mathrm{GL}(n, K)$ the group of matrices in $M(n, K)$ with nonzero determinant, and $K[x_1, \cdots, x_n]$ the algebra of polynomials in n variables with coefficients in K.

2. Algebraic groups.

A subset V of K^n is called an algebraic set if there exists an ideal $j(V) \subseteq K[x_1 \cdots x_n]$ such that

$$V = \{p \in K^n : f(p) = 0 \quad \text{for all } f \in j(V)\}.$$

The ideal $j(V)$ is called the ideal of V. If I is an ideal in $K[x_1, \cdots, x_n]$ then the ideal $\mathrm{rad}(I) = \{f \in K[x_1, \cdots, x_n] : f^r \in I$ for some non-negative integer $r\}$ is called the radical of I. We say V is an irreducible algebraic set if $\mathrm{rad}\, j(G) = j(G)$. If $p \in V$ then $M_p = \{f \in K[x_1 \cdots x_n] : f(p) = 0\}$ is a maximal ideal and if $p = (a_1, \cdots, a_n)$ we have $M_p =$