PACIFIC JOURNAL OF MATHEMATICS
Vol. 70, No. 2, 1977

COUNTABLE SPACES WITHOUT POINTS OF FIRST COUNTABILITY

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In this paper we show that there are 2^c non-homeomorphic countable regular spaces, each of which has no point of first countability. Several specific countable regular spaces are shown not to be homeomorphic.

1. Preliminaries. A countable space need not be first countable. One example of such a space is $N \cup \{p\}$ where $p \in \beta N - N$ and the topology is the relative topology of βN. This space, however, has many points of first countability—indeed all of the points of N are isolated. Several examples of countable spaces without points of first countability are known to exist.

N denotes the space of natural numbers including 0, Q denotes the space of rational numbers, and $\mathbf{R}$ denotes the space of reals. The cardinal of $\mathbf{R}$ is denoted c. If X is a completely regular Hausdorff space, βX is the Stone–Cech compactification of X. If X and Y are spaces and $f: X \to Y$ is a continuous surjection, f is *irreducible* if there is no proper closed subset K of X such that $f(K) = Y$. It is well-known (see for example **[11]**, 10.48) that if X and Y are compact Hausdorff spaces and $f: X \to Y$ is a continuous surjection, there is a closed subset K of X such that $f(K) = Y$ and the restriction of f to K is irreducible. A space X is *resolvable* if X contains disjoint dense subsets. A space X is *homogeneous* if for any pair of points p, $q \in X$, there is a homeomorphism $f: X \to X$ such that $f(p) = q$. A *rigid* space is a space whose only auto-homeomorphism is the identity.

For $n \in N$, let R_n be $N^{\{1,\cdots,n\}}$. Since for $n = 0$, $\{1,\cdots,n\} = \varnothing$, $R_0 = \{\varnothing\}$. The empty set, when viewed as the element of R_0, is denoted p_0. Let $S = \bigcup_{n \in N} R_n$ and define an order $\leqq$ on S by $p \leqq q$ if and only if $p \in R_m$, $q \in R_n$ with $m \leqq n$ and $q \mid \{1, \cdots, m\} = p$. $(S, \leqq)$ is a tree (see **[6]**) and is clearly countably infinite. For $x \in S$, A_x is the set $\{p \in S: x \leqq p, x \neq p$, and $x \leqq y \leqq p$ implies $x = y$ or $y = p\}$; thus, A_x is the set of immediate successors of x. For $x \in S$, $U \subseteq N$, let $K_x^U = \{x\} \cup \{p \in S$: There is a $q \in A_x$ such that the last entry of q is an element of U and $q \leqq p\}$.

If $p \in \beta N - N$, that is, p is a free ultrafilter on N, then Σ_p denotes the subspace $N \cup \{p\}$ of βN. Two points p and q of $\beta N - N$ are the *same βN-type*, or simply the *same type*, if Σ_p is homeomorphic to Σ_q, or,