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A CHARACTERIZATION FOR COMPACT CENTRAL DOUBLE CENTRALIZERS ON C^*-ALGEBRAS

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The purpose of this note is to give a characterization for compact central double centralizers on any C^*-algebra A in view of the Dixmier's representation theorem of central double centralizers on A. The proof makes use of the Urysohn's lemma for spectra of C^*-algebras and algebraic properties of a central double centralizer.

Throughout the note, A denotes a C^*-algebra. Let $\mathrm{Prim}\,A$ denote the structure space of A, that is the set of all primitive ideals of A, with the hull-kernel topology. Let $M(A)$ denote the double centralizer algebra of A and $Z(M(A))$ the center of $M(A)$. Busby [1] has noted that the algebra $C^b(\mathrm{Prim}\,A)$ of all bounded continuous complex-valued functions on Prim A can be canonically identified with $Z(M(A))$, which is equivalent with a result of Dixmier ([5], Theorem 5). Moreover, we can regard the algebra $Z(M(A))$ as the algebra of all bounded linear operators T on A such that $(Tx)y = x(Ty)$ for all $x, y \in A$. In its final form, this identification Φ between $Z(M(A))$ and $C^b(\mathrm{Prim}\,A)$ can be described as follows: If $T \in Z(M(A))$, then $Ta + P = \Phi(T)(P)(a + P)$ for all $a \in A$ and $P \in \mathrm{Prim}\ A$, where $a + P$ for $P \in \mathrm{Prim}\ A$ denotes the canonical image of a in A/P (Dauns and Hofmann theorem [3] shows that every functions in $C^b(\mathrm{Prim}\,A)$ can be realized uniquely in this way). We will characterize the set of all compact central double centralizers on A in view of this representation theorem of $Z(M(A))$. Our characterization is similar to ones established by Kellogg [6] and Ching and Wong [2] for H^*-algebras, and this is also a generalization of one proved by Rowlands [7] for dual B^*-algebras.

Let $Z_C(M(A))$ denote the compact central double centralizers on A. If $LC(A)$ is the algebra of all compact operators on A, then $Z_C(M(A)) = Z(M(A)) \cap LC(A)$, so that $Z_C(M(A))$ is a closed ideal of $Z(M(A))$. Let I_C be the set of all functions f in $C^b(\mathrm{Prim}\,A)$ such that for any closed compact subset K in $\mathrm{supp}\,(f)$, A/I_K is finite dimensional. Here $\mathrm{supp}\,(f)$ denotes the set of all $P \in \mathrm{Prim}\,A$ such that $f(P) \neq 0$, and I_K denotes a closed two-sided ideal of A with $\mathrm{Prim}\,(A/I_K) \simeq K$ (cf. [4], §3.2). Note that if K is the empty set, then A/I_K is zero-dimensional, so that I_C contains the zero function. Now I_C is a closed ideal in $C^b(\mathrm{Prim}\,A)$. For since $\mathrm{supp}\,(f) \supset \mathrm{supp}\,(fg)$ for each f, g in $C^b(\mathrm{Prim}\,A)$, I_C is an ideal in $C^b(\mathrm{Prim}\,A)$. Let $\{f_n\}$ be a sequence of functions in I_C which converges uniformly