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CHARACTERS OF P'-DEGREE IN SOLVABLE GROUPS

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We prove that $|I_p(G)| = |I_p(N(P))|$ for $P \in \mathrm{Syl}\,(G)$, for solvable G. Here p is a prime and $I_p(G)$ is the set of irreducible characters ψ such that $(\psi(1), p) = 1$.

1. Introduction. The groups considered are finite and the group characters are defined over the complex numbers. McKay conjectured $|I_p(G)| = |I_p(N(P))|$ where $P \in \mathrm{Syl}\,(G)$ for simple G and $p = 2$ [6]. I. M. Isaacs has proven the result when $|G|$ is odd and p is any prime (Theorem 10.9 of [4]). We prove the result for solvable G. In fact we generalize this slightly to sets of primes and normalizers of Hall subgroups.

For characters χ and ψ of G, we let $[\chi, \psi]$ denote the inner product of χ and ψ. Let $N \trianglelefteq G$ and $\theta \in IRR(N)$. We write $I_G(\theta)$ to denote the inertia group $\{g \in G \mid \theta^g = \theta\}$. We also write $IRR(G \mid \theta) = \{\chi \in IRR(G) \mid [\chi_N, \theta] \neq 0\}$. Of course, character induction yields a one-to-one map from $IRR(I_G(\theta) \mid \theta)$ onto $IRR(G \mid \theta)$. If $\chi \in IRR(G \mid \theta)$; we say χ (or θ) is fully ramified with respect to G/N if $\chi_N = e\theta$ and $e^2 = |G: N|$. This will occur if $I_G(\theta) = G$ and χ vanishes off N.

Suppose that K/L is an abelian chief factor of G; $\gamma \in IRR(K)$; $\phi \in IRR(L)$; and $[\gamma_L, \phi] \neq 0$. If $K \cdot I_G(\phi) = G$, then one of the following occur:

(a) $\gamma_L = \phi$;

(b) γ and ϕ are fully ramified with respect to K/L, or

(c) $\phi^K = \gamma$.

We note that $K \cdot I_G(\phi) = G$ whenever $I_G(\gamma) = G$. The results of these last two paragraphs are well known (e.g. see Chapter 6 of [5]); and we will use them without reference. In Theorem 3.3, we use known results about character triple isomorphisms (see §8 of [4] or Chapter 11 of [5]); otherwise, everything should be self-explanatory.

I would like to thank E. C. Dade for his preprint [1].

2. Extendability. A straightforward proof of Lemma 2.1 may be found in Lemma 10.5 of [4].

LEMMA 2.1. *Assume $N \trianglelefteq G$, $H \leqq G$, $NH = G$, and $N \cap H = M$. Assume $\phi \in IRR(N)$ is invariant in G and $\phi_M \in IRR(M)$. Then $\chi \leftrightarrow \chi_H$ defines a one-to-one correspondence between $IRR(G \mid \phi)$ and $IRR(H \mid \phi_M)$.*