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METABELIAN GROUPS WITH AN IRREDUCIBLE PROJECTIVE REPRESENTATION OF LARGE DEGREE

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The structure of a metabelian group admitting an irreducible projective complex representation whose degree is the square root of the order of the group studied.

In [7] a class of metabelian groups admitting a faithful irreducible projective representation was characterized. If $\bar{G}$ is a finite group with irreducible projective representation T^* so that $T^*(1)^2 = [\bar{G}:1]$ then T^* is a faithful irreducible projective representation. Conversely, if G is abelian and T^* is a faithful irreducible projective representation then $T^*(1)^2 = [\bar{G}:1]$. We study metabelian groups $\bar{G}$ admitting an irreducible projective representation T^* with $T^*(1)^2 = [\bar{G}:1]$. Given such a group one can construct, using Schur's theory, a central extension G of $\bar{G}$ with center Z and an ordinary faithful irreducible representation T of G with $T(1)^2 = [G:Z]$ and $T^*(gZ) = T(g)$ for all $g \in G$ (see p. 358 of [1]). A group G with an irreducible representation T so that $T(1)^2 = [G:Z]$ is called a group of central type.

Groups of central type have been studied in several places (see for example [3] and [5]). We study metabelian groups $\bar{G}$ admitting an irreducible projective complex representation whose degree is $\sqrt{[\bar{G}:1]}$ by studying the associated central extension G of central type using the methods of ordinary representation theory. In this note all groups are finite and all representations and characters are assumed to be taken over the complex numbers. All unexplained terminology and notation is as in [1].

I. An abelian group G is said to be of symmetric type in case $G = A \times A$ for some abelian group A. It has been shown that an abelian group G is a group of symmetric type if and only if G admits a faithful irreducible projective representation [4] if and only if G admits an irreducible projective representation T^* with $T^*(1)^2 = [G:1]$([2]). For any group H let $Z(H)$ denote the center of H.

THEOREM 1. *Let G be a group of central type with center Z. Assume there is a faithful irreducible character ζ on G with $\zeta(1)^2 = [G:Z]$. If there is a normal subgroup H of G with $Z \subseteqq H \subseteqq G$*