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A HOMEOMORPHISM CLASSIFICATION OF WILDLY IMBEDDED TWO-SPHERES IN S^3

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If a two-manifold M is wildly imbedded S^3, it gives rise to a pair of noncompact three-manifolds of a special type. This type is considered in detail and a homeomorphism classification theorem for it is derived. This result is then used to decide whether there is a homeomorphism of S^3 to itself, which takes M onto another, given two-manifold.

Introduction. In recent years much work has been done on wild imbeddings of two-spheres in three-spheres. The survey article [5] gives an excellent bibliography of this work. Relatively little of this work, however, involves the homotopy type of such imbeddings or algebraic characterizations of them. Two papers which make some use of these are [3] and [6]. In this paper we develop some of this theory. An imbedding of S^2 in S^3 gives rise to a pair of noncompact three-manifolds with boundary. If these three-manifolds are trail irreducible (this term will be defined in §1) and the set of wild points of S^2 is totally disconnected, we can develop a useful uniform homotopy theory which allows us to determine the homeomorphism type of the manifold from the homotopy type. Homeomorphisms of the three-manifolds may then be sewn together to form a homeomorphism theorem for the imbedding. Although the case of two-spheres imbedded in S^3 is historically of greatest interest, there is nothing restricting our methods to that case, so we shall develop the theory for finitely many closed two-manifolds imbedded in S^3. We also derive a method for showing that an imbedding is trail irreducible. Using this method we show the existence of uncountably many distinct imbeddings for which the theorems hold. In particular we show that they hold for Alexander's horned sphere.

1. Notation and definitions. Suppose $M_1, \cdots, M_k$ are closed two-manifolds and that they are disjointly (and possibly wildly) imbedded in S^3. Let $M = M_1 \cup \cdots \cup M_k$. Then $S^3 - M$ has $k+1$ components. Let T be one component. Let $I = \{(x, y, z) \in R^3: z \geqq 0\}$.

DEFINITION. A point p of $M \cap \bar{T}$ is *tame from* T if there is a homeomorphism $h: (I, \partial I) \to (T, M)$ with $p \in h(\partial I)$. A point of $M \cap \bar{T}$ is *wild from* T if it is not tame from T.