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ON THE STRONG COMPACT-PORTED TOPOLOGY FOR SPACES OF HOLOMORPHIC MAPPINGS

M. BIANCHINI, O. W. PAQUES AND M. C. ZAINE

Suppose E is a separated complex locally convex space, U is non void open subset of E, F a complex normed space and $\mathscr{H}(U; F)$ the complex vector space of all holomorphic mappings from U into F. On $\mathscr{H}(U; F)$ we consider the following topologies; a) $\tau_{\omega s}$, the topology generated by the seminorms p which are $K-B$ ported for some $K \subset U$ compact and $B \subset E$ bounded. A seminorm p is $K-B$ ported if for every $\varepsilon > 0$, with $K + \varepsilon B \subset U$, there is $c(\varepsilon) > 0$, such that $p(f) \leqq c(\varepsilon) \sup\{\|f(x)\|; x \in K + \varepsilon B\}$ for all $f \in \mathscr{H}(U; F)$; b) τ_0, the compact open topology; c) $\tau_{\infty s}$ the topology defined by J. A. Barroso in "Topologias nos espaços de aplicações holomorfas entre espaços localmente convexos", An. Acad. Brasil. Ci, 43, 1971. The topology $\tau_{\omega s}$ is an generalization of the Nachbin topology (L. Nachbin, Topology on Spaces of Holomorphic Mappings, Springer-Verlag, 1968). The following results are valid: 1. $\mathscr{X} \subset \mathscr{H}(U; F)$ is τ_0-bounded if, and only if, $\mathscr{X}$ is $\tau_{\omega s}$-bounded. 2. $\mathscr{X} \subset \mathscr{H}(U; F)$ is $\tau_{\omega s}$-relatively compact if, and only if, $\mathscr{X}$ is $\tau_{\infty s}$-relatively compact. 3. Let E be a quasi complete space. Then $\tau_0 = \tau_{\omega s}$ on $\mathscr{H}(E; C)$ if, and only if E is a semi-Montel space. Moreover, the completion of $\mathscr{H}(E; C)$ on the $\tau_{\omega s}$ topology and the bornological topology associated to τ_0 are caracterized via the Silva-holomorphic mappings.

Throughout this article the following notations will be used. E is a complex separated locally convex space; U is a non void open subset of E; F is a complex normed space; $\mathscr{H}(U; F)$ is the complex vector space of all holomorphic mappings from U into F: $\mathscr{P}(^nE; F)$ is the complex vector space of all continuous n-homogeneous polynomials from E into F; $(1/n!)\hat{d}^n f(t) \in \mathscr{P}(^nE; F)$ is the nth coefficient of the Taylor series of f at $t, n = 0, 1, \cdots, f \in \mathscr{H}(U; F)$; τ_0 is the compact open topology on $\mathscr{H}(U; F)$; $\tau_{\infty s}$ is the locally convex topology on $\mathscr{H}(U; F)$ generated by all seminorms of the type

$$p_{K,n,B}(f) = \sup\left\{\left\|\frac{1}{n!}\,\hat{d}^n f(t)u\right\|; t \in K,\ u \in B\right\}$$

where $n = 0, 1, \cdots, K$ is a compact subset of U, B is a bounded balanced subset of E; $\mathscr{P}_s(^nE; F)$ is $\mathscr{P}(^nE; F)$ endowed with the locally convex topology of the uniform convergence on bounded subsets of E. We will introduce a new locally convex topology, $\tau_{\omega s}$, on $\mathscr{H}(U; F)$ which, in some cases, coincides with the Nachbin