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ON THE CHARACTERIZATIONS OF THE BREAKDOWN POINTS OF QUASILINEAR WAVE EQUATIONS

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We consider the mixed initial and boundary value problem of the quasilinear wave equation:

$$(1)\qquad \begin{aligned} u_t - v_x &= 0\,, \\ v_t - Q^2(u)u_x &= 0\,; \end{aligned}$$

$$(2)\qquad \begin{aligned} &u(x, 0) = 0,\, v(x, 0) = v_0(x),\, 0 \leqq x \leqq 1\,, \\ &v(0, t) = v(1, t) = 0,\, t \geqq 0\,. \end{aligned}$$

In general the solution of the system (1), (2) eventually breaks down in the sense that some of its first derivatives become unbounded at a finite time. It is shown that there are only finitely many breakdown points and that at each of them there originates one or two shock curves.

The fact that solutions of (1), (2) eventually break down has been derived by many authors, e.g., see [12], [9], [5], [10], [6], [7], and [1]. It should be pointed out however, that even though a solution break down in finite time it can be extended to large t as a weak solution. See e.g., [3], [4], and [11].

We define $r = v + M(u)$ and $s = v - M(u)$, where $M(\xi) = \int_0^\xi Q(\zeta)d\zeta$. Then r and s are Riemann invariants of (1). Let $q(\eta) = Q(M^{-1}(\eta/2))$. After transforming (1) to equations with r and s as dependent variables we apply a hodograph transformation to invert the resulting equations to

$$(3)\qquad \begin{aligned} x_r - q(r - s)t_r &= 0\,, \\ x_s + q(r - s)t_s &= 0\,. \end{aligned}$$

Eliminating x in (3) gives

$$(4)\qquad t_{rs} = \rho(r - s)(t_r - t_s)$$

where $\rho(\eta) = Q'(M^{-1}(\eta/2))/4q^2(\eta)$.

We assume that

(5) $Q(\xi)$ is a positive analytic function over $(-\infty, \infty)$;

(6) $v_0(x) = f|_{[0,1]}(x)$ is concave over $[0, 1]$, where $f(x)$ is an odd periodic analytic function over $(-\infty, \infty)$ with period 2.

Let $a = \max_{0 \leqq x \leqq 1} f(x) = f(b)(0 < b < 1)$, $\Omega = \{(r, s): |r| \leqq a; |s| \leqq a\}$, $\Omega_i = \{(r, s) \in \Omega: (-1)^{i-1}(r - s) \geqq 0\}(i = 1, 2)$, $a_1 = M(\infty)$ and $a_2 = -M(-\infty)$. We assume further that