

A NOTE ON THE AUTOMORPHISM GROUPS OF SIMPLE DIMENSION GROUPS

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We study the automorphism of simple dimension groups with underlying group Z^2 (which are of the form (Z^2, P_α) , α a positive irrational) and find out the necessary and sufficient condition for the dimension group (Z^2, P_α) having nontrivial automorphisms is α being a quadratic surd. In this case we also obtain the classification of the automorphism via the Bratteli diagram of the dimension groups.

Introduction. In his paper [6], George Elliott proposed the notion of dimension groups for the approximately finite C^* -algebras (AF algebras) which were originally studied by J. Glimm, J. Dixmier, and O. Bratteli [1]. He showed that the classification of the dimension groups is equivalent to the stable classification of AF algebras. He also found that the dimension groups satisfy the *Riesz interpolation property* (see §1) and raised the problem of intrinsic characterization of dimension groups. An ordered group (see §1) which satisfies the Riesz interpolation property is called a *Riesz group*. It is shown by Effros, Handelman, and the author [5] that Riesz groups are in fact dimension groups.

Although we know that Riesz groups are dimension groups, for the practical applications, we still have to have a method for finding the corresponding Bratteli diagram [1]. In [3], [4], we have studied this problem for certain classes of dimension groups. On the other hand, it is also important to understand the structure of the automorphisms of AF algebras and the relation of those to the automorphisms of the corresponding dimension groups. This question has been studied by Elliott [6], Effros and Rosenberg [2] (also see [5]). By their results, we see that automorphism of dimension groups tells us a certain amount of information about the approximately inner automorphisms of AF algebras.

In this note, we study the automorphism groups of the dimension groups of the form (Z^2, P_α) (see §1 for definition) where α is irrational. In this case, the continued fraction analysis of [3] makes theory particularly transparent. We also provide some information about the automorphisms of higher rank simple dimension groups. In §2, we give a necessary and sufficient condition for when the automorphism group is nontrivial, we also obtain the classification of the automorphisms via the Bratteli diagram of the dimension groups. As pointed out in [6], [5], the automorphism groups of