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A NOTE ON DISCONJUGACY FOR SECOND ORDER SYSTEMS

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It is well-known that the equation

$$x''+A(t)x=0 \tag{1}$$

is disconjugate on $[a, b]$ if and only if there exists a solution which is positive on $[a, b]$, in the case that $A(t)$ is scalar-valued. In this note we generalize this simple result to the case where $A(t)=(a_{ij}(t))$ is an $n\times n$ matrix-valued function which satisfies certain generalized sign conditions. These results apply, for instance, if the off diagonal elements are nonnegative. Simple necessary and sufficient conditions are given for disconjugacy if $A(t)\equiv A$ and these are used to construct examples showing the necessity of sign conditions on $A(t)$ for the above mentioned results and other results of Sturm type for systems to be valid.

Introduction. Many authors have considered the problem of extending the well-known results on disconjugacy for the scalar equation (1) to systems. We mention the work of Morse [8] and Hartman and Wintner [5], where $A(t)$ is assumed symmetric or conditions are placed on the symmetric part of A. Recently, many new results have been obtained in the papers of Ahmad and Lazer ([1], [2], [3]) and Schmitt and the author, [9], where symmetry assumptions have generally been avoided.

Recall that (1) is said to be disconjugate on the interval $[a, b]$ if no nontrivial solution of (1) vanishes twice on $[a, b]$, otherwise (1) is conjugate on $[a, b]$. If $x\in R^n$, we write $x\geqq 0$ if $x_i\geqq 0$, $1\leqq i\leqq n$; $x>0$ if $x\geqq 0$ and $x\neq 0$; and $x\gg 0$ if $x_i>0$, $1\leqq i\leqq n$. If A is an $n\times n$ matrix we denote by $\sigma(A)$ the spectrum of A.

Below we state two corollaries of our main results and some examples to indicate the necessity of the hypotheses involved. The main results are stated in § 2 and the proofs are given in § 3.

COROLLARY 1. *Let $A(t)=(a_{ij}(t))$ be a continuous, matrix-valued function satisfying $a_{ij}(t)\geqq 0$, $i\neq j$. If (1) is disconjugate on $[a, b]$ then there is a solution $x(t)$ of (1) satisfying $x(t)>0$ on $[a, b]$.*

COROLLARY 2. *Let $A(t)$ satisfy the conditions of Corollary 1. If there exists a solution $y(t)$ of the differential inequality $y''+A(t)y\leqq 0$ satisfying $y(t)\gg 0$, $a\leqq t\leqq b$, then (1) is disconjugate on $[a, b]$.*