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LATTICES WITH UNIQUE COMPLEMENTATION

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R. P. Dilworth's theorem that every lattice is a sublattice of a uniquely complemented lattice is shown to hold in $2^{\aleph_0}$ varieties of lattices.

1. Introduction. After E. V. Huntington [11], it was conjectured that every uniquely complemented lattice was a distributive lattice; since a uniquely complemented distributive lattice is a Boolean lattice and every Boolean lattice is uniquely complemented, the verification of such a conjecture would have provided a characterization of Boolean lattices. For a uniquely complemented lattice L, G. Birkhoff and J. von Neumann showed that if L is modular or relatively complemented then L is distributive. Subsequently, G. Birkhoff and M. Ward [4] showed that if L is complete, atomic, and dually atomic then L is distributive. Further, R. P. Dilworth [6] verified that if L is finite dimensional then it is distributive. Finally, the conjecture was refuted in, the now famous paper, [7]; R. P. Dilworth proved that every lattice is a sublattice of a uniquely complemented lattice (see also C. C. Chen and G. Grätzer [5]). Since then a number of other sufficient conditions for distributivity of a uniquely complemented lattice have been discovered. For example, if a uniquely complemented lattice L is either atomic (T. Ogasawara and U. Sasaki [15], and J. E. McLaughlin [14]), algebraic (V. N. Saliĭ [17]), or if the function that sends $l \in L$ to the unique complement of l is order inverting (G. Birkhoff [3]), then L is distributive.

The lattices constructed by R. P. Dilworth in [7] contain the free lattice on countably many generators as a sublattice. Hence, in particular, any nontrivial lattice identity fails to hold in any of Dilworth's lattices. (By a nontrivial identity, we mean an identity that does not follow from the lattice axioms.) A growing conjecture has been that any uniquely complemented lattice that satisfies a nontrivial lattice identity is distributive. In this connection (see G. Grätzer [9]), R. Padmanabhan [16] has shown that a uniquely complemented lattice in the variety $\boldsymbol{M} \vee \boldsymbol{N}_5$, or in the variety generated by a finite lattice satisfying one of two implications (namely, $(SD_\wedge)$ or an implication due to E. Fried and G. Grätzer, [9]) is distributive. However, we will show that this is not indicative of the general situation; that is to say, we will show that there are $2^{\aleph_0}$ varieties of lattices for which Dilworth's theorem holds. Thus, we will prove: