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ON COMPACT METRIC SPACES WITH NONCOINCIDING TRANSFINITE DIMENSIONS

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For every no more than countable ordinal number α we shall define an ordinal number $\varphi(\alpha)$ such that for every compact metric space X with $\operatorname{ind} X \leq \alpha$ we have $\operatorname{Ind} X \leq \varphi(\alpha)$ and there exists a compact metric spaces X_α with $\operatorname{ind} X_\alpha = \alpha$, $\operatorname{Ind} X_\alpha = \varphi(\alpha)$, where $\operatorname{ind} X_\alpha$ and $\operatorname{Ind} X_\alpha$ mean small and large transfinite inductive dimensions respectively. In particular we now extend the author's previous result on existence of compact metric spaces with noncoinciding transfinite dimensions.

1. Introduction. In this paper we consider only metric spaces. For instance, by a compact space we mean a compact metric space. All mappings we consider are continuous and I^n denotes the n-dimensional euclidean cube.

1. Definitions and statements of main results.

DEFINITION 1.1. (a) $\operatorname{ind} X = -1 \Leftrightarrow X = \varnothing$.

(b) We assume that for every ordinal number $\alpha < \beta$ the class of spaces X with $\operatorname{ind} X \leqq \alpha$ is defined. Then, we say $\operatorname{ind} X \leqq \beta$ if for every point $x \in X$ and a closed subset F, $x \notin F \subset X$, there exists a neighborhood Ox of x such that:

$$Ox \subset X \backslash F$$

$$\operatorname{ind} FrOx \leqq \alpha < \beta^{1}$$

We put $\operatorname{ind} X = \min\{\beta: \operatorname{ind} X \leqq \beta\}$.

(c) We say that dimension $\operatorname{ind}_x X$ of a space X in a point $x \in X \leqq \beta$ if there exists such a base $\{O_\lambda: \lambda \in A\}$ at this point, so that

$$\operatorname{ind} FrO_\lambda < \beta .$$

We put $\operatorname{ind}_x X = \min\{\beta: \operatorname{ind} X \leqq \beta\}$.

DEFINITION 1.2. (a) $\operatorname{Ind} X = -1 \Leftrightarrow X = \varnothing$

(b) Let, for every ordinal number $\alpha < \beta$, the class of spaces X with $\operatorname{Ind} X \leqq \alpha$ be defined. Then, $\operatorname{Ind} X \leqq \beta$ if for every pair of disjoint closed subsets F and G there exists a partition C[2]

[1] Fr A denotes the boundary of A.

[2] By a partition in X between sets A and B we mean a closed set C in X such that $X \backslash C = U \cup V$, $U \cap V = \varnothing$, $A \subset U$, $B \subset V$ for some open sets U and V in X.