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TENSOR PRODUCTS OF BANACH BUNDLES

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This paper is concerned with projective and inductive tensor products of bundles of Banach spaces. Let $\pi: E \to S$ and $\rho: F \to T$ be bundles of Banach spaces over the locally compact Hausdorff spaces S and T, with fibers $\{E_s: s \in S\}$ and $\{F_t: t \in T\}$, respectively. Let $\Gamma_0(\pi)$ and $\Gamma_0(\rho)$ be their spaces of sections which disappear at infinity. We show the existence of a bundle $\pi \hat{\otimes} \rho: E \hat{\otimes} F \to S \times T$ whose fibers are $\{E_s \hat{\otimes} E_t: (s, t) \in S \times T\}$; if $\sigma \in \Gamma_0(\pi)$ and $\tau \in \Gamma_0(\rho)$, then their pointwise tensor $\sigma \odot \tau$ defined by $(\sigma \odot \tau)(s, t) = \sigma(s) \otimes \tau(t)$ is a section of the bundle $\pi \hat{\otimes} \rho: E \hat{\otimes} F \to S \times T$. Further, we show the existence of a bundle $\pi \hat{\hat{\otimes}} \rho: E \hat{\hat{\otimes}} F \to S \times T$ whose fibers are $\{E_s \hat{\hat{\otimes}} F_t: (s, t) \in S \times T\}$, and demonstrate that $\Gamma_0(\pi) \hat{\hat{\otimes}} \Gamma_0(\rho)$ and $\Gamma_0(\pi \hat{\hat{\otimes}} \rho)$ are isometrically isomorphic.

The present paper continues the study begun in [5] of the relationships between Banach modules and bundles of Banach spaces. Specifically, it concerns tensor products of such objects.

Given two bundles of Banach spaces $\pi: E \to S$ and $\rho: F \to T$ having locally compact base spaces we show that there is a bundle of Banach spaces $\theta: G \to S \times T$ having the following properties:

(1) for each pair (s, t) in $S \times T$ the stalk $G_{st} = \theta^{-1}(\{(s, t)\})$ is $E_s \hat{\otimes} F_t$, where, as in [5], $E_s = \pi^{-1}(\{s\})$ and $F_t = \rho^{-1}(\{t\})$;

(2) given two sections $\sigma \in \Gamma_0(\pi)$ and $\tau \in \Gamma_0(\rho)$ their pointwise tensor product $\sigma \odot \tau$ defined by

$$(\sigma \odot \tau)(s, t) = \sigma(s) \otimes \tau(t)$$

is a section of the bundle $\theta: G \to S \times T$. (Here again the reader is referred to [5] for notation and terminology.)

The bundle $\theta: G \to S \times T$ is called the *projective tensor product* of the given bundles and is denoted by $\pi \hat{\otimes} \rho: E \hat{\otimes} F \to S \times T$.

Tensor products of Banach bundles relate to tensor products of Banach modules in the following fashion. Suppose that A and B are commutative Banach algebras with maximal ideals spaces S and T. Suppose further that (M, A) and (N, B) are Banach modules which satisfy the (KR) condition. Then

(1) the Banach module $(M \hat{\otimes} N, A \hat{\otimes} B)$ also satisfies the (KR) condition;

(2) the canonical bundle associated with $(M \hat{\otimes} N, A \hat{\otimes} B)$ is (bundle isomorphic to) the tensor product of the canonical bundles