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# A CHARACTERIZATION OF LOCALLY MACAULAY COMPLETIONS

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**The purpose of this note is to prove the following theorem.**

THEOREM 1.1. **Let $(R, m)$ be a Noetherian local ring of dimension $d \geqq 1$ and depth $d-1$. By $\hat{R}$ denote the completion of $R$ in the $m$-adic topology. Then the following are equivalent:**

**(1) $\hat{R}$ is equidimensional and satisfies Serre's property $S_{d-1}$**

**(2) $H_m^{d-1}(R)$ has finite length**

**(3) There exists an $N > 0$ such that if $x_1, \cdots, x_d$ is a sequence of elements $R$ with $\mathrm{ht}(x_{i_1}, \cdots, x_{i_j}) = j$ for all $j$-elements subsets of $\{1, \cdots, n\}$, $1 \leqq j \leqq n$, and if $m_i \geqq N$, $1 \leqq i \leqq d$, then $x_1^{m_1}, \cdots, x_d^{m_d}$ is an unconditioned $d$-sequence.**

Recall the local ring $(S, N)$ is *equidimensional* if for every minimal prime divisor $p$ of zero, $\dim S/p = \dim S$.

Serre's property $S_k$ is that

$$\operatorname{depth} R_p \geqq \min [\mathrm{ht}\, p, k]$$

for all primes $p$.

We will always denote the local cohomology functor by $H_m^j(\_\_)$ ([1]).

We recall the definition of a $d$-sequence due to this author [3].

DEFINITION 0.1. A system of elements $x_1, \cdots, x_d$ in a commutative ring $R$ is said to be a $d$-sequence if

(1) $x_i \notin (x_1, \cdots, \hat{x}_i, \cdots, x_d)$

(2) $((x_1, \cdots, x_i): x_{i+1}x_k) = ((x_1, \cdots, x_i): x_k)$ for $k \geqq i+1$ and $i \geqq 0$.

A $d$-sequence is said to be unconditioned if any permutation of it remains a $d$-sequence.

These have been studied extensively by this author and have been useful to determine the "analytic" properties of ideals generated by them. In [3] the following was skown:

PROPOSITION. *Let $(R, m)$ be a local Noetherian ring. Then $R$ is Buchsbaum (see [10] for a definition and discussion) if and only if every system of parameters forms a $d$-sequence.*

Thus Theorem 1.1 may be seen as a related result, characterizing rings in which "almost all" s.o.p.'s form a $d$-sequence. Independent