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REPRESENTATIONS OF HOMOLOGY 3-SPHERES

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Every homology 3-sphere can be represented as a framed link, in the sense of Kirby, of the following special type. There are k disjoint embeddings in S^3 of a genus one surface with two boundary components. The link is the $2k$ boundary components. If q is the linking number of the ith pair, then one of the components has framing $q+1$, the other $q-1$.

1. Introduction. A very useful and fairly recent method for studying 3-manifolds, in particular for the study of examples, is R. Kirby's "Calculus of framed links" (see [8], [7]). Another, older, method is through the mapping class group of a surface via Heegaard splittings (see [1], [4]). Through the work of Birman [2], Powell [9], and Johnson [6], a geometrically appealing (but infinite) set of generators has been found for group of homeomorphisms of an orientable surface, up to isotopy, that induce the identity on homology. It is the purpose of this paper to use this set of generators to obtain a representation theorem for all homology 3-spheres as special framed links. It turns out that the formula for the μ-invariant of homology 3-spheres represented this way is particularly simple.

2. Notation and conventions. Throughout the paper X_g will be a 3-dimensional genus g handlebody, T_g its boundary, X_g' another such handlebody, and i a PL homeomorphism from T_g to $\partial X_g'$ so that $X_g \cup X_g'$ defines a Heegaard splitting of S^3. Also, A will be an "annulus with a handle"; that is an oriented genus one surface with two boundary components. The boundary components derive orientations from the orientation of A and will be denoted a and b. We assume the reader is familiar with such concepts as "characteristic surface", "intersection matrix", "index", and "μ-invariant" as they apply to three and four manifolds. These terms are defined in [8].

3. Several propositions. In this section we state several results needed for the proof of the main theorem.

PROPOSITION 1. *Let H^3 be a genus g homology 3-sphere. There is a homeomorphism ϕ of T_g such that ϕ induces the identity on the homology of T_g and $H^3 = X_g \bigcup_{i\phi} X_g'$.*