PACIFIC JOURNAL OF MATHEMATICS
Vol. 95, No. 2, 1981

FINITE SIGNED MEASURES ON FUNCTION SPACES

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Some results for probability measures on function spaces are extended to finite signed measures (FSM's). In particular FSM's on the space of continuous functions and right-continuous functions with left-hand limits are patched together by a procedure of Stroock and Varadhan. Given an increasing sequence of stopping times the procedure is carried out repeatedly. A sequence of transition functions and, an extension result for the linear maps associated with these transition functions are obtained.

Introduction. In recent years some papers have appeared related to signed measures on function spaces (see [3] and [4]). This paper extends certain results for probability measures on function spaces to finite signed measures (FSM's) on such spaces. A more detailed discussion can be found in [8].

In part I we introduce conditional FSM's and consider the existence of a regular conditional distribution (RCD) of an FSM on a standard measurable space mimicking Chapter V of [5]. Further, the Jordan decomposition of an RCD is investigated. We then consider a sequence of transition functions and associate linear maps between Banach spaces of FSM's with these transition functions. An extension result for the linear maps is obtained.

In part II FSM's on $\Omega = C([0, \infty); S)$ and $\Omega = D([0, \infty); S)$ with S a separable metric space are patched together by the procedure used in [6] for probability measures on $C([0, \infty); R^d)$. In fact, if $\mathscr{M}^s$ is the σ-field on Ω generated by the coordinate projections $\{X_t,\ t \geqq s\}$ and τ an s-stopping time with respect to the σ-fields $\mathscr{M}_t^s = \sigma\{X_u,\ s \leqq u \leqq t\}$, then an FSM on $\mathscr{M}_\tau^s$ is patched together with a family $\{\mu_\omega\}_{\omega \in \Omega}$ of FSM's where μ_ω has domain $\mathscr{M}^{\tau(\omega)}$ if $\tau(\omega) < \infty$.

If S is a complete separable metric space, $(\Omega, \mathscr{M}^s)$ and $(\Omega, \mathscr{M}_\tau^s)$ are standard measurable spaces. In this case any FSM on $(\Omega, \mathscr{M}^s)$ with an RCD given $\mathscr{M}_\tau^s$ can be thought of as obtained by patching. If $\tau_0, \tau_1, \cdots$ is an increasing sequence of s-stopping times $\mathscr{M}_{\tau_0}^s$, $\mathscr{M}_{\tau_1}^s, \cdots$ is the corresponding sequence of σ-fields and, given families of FSM's $\{\mu_{n\omega}\}_\omega$ on $\mathscr{M}^{\tau_n(\omega)}$ for each n with the right properties the patching procedure can be applied repeatedly. We have in fact an associated sequence of transition functions and the results of part I apply.

Basic facts on FSM's are taken from [2].