PACIFIC JOURNAL OF MATHEMATICS
Vol. 96, No. 1, 1981

TOPOLOGICAL METHODS FOR C^*-ALGEBRAS I: SPECTRAL SEQUENCES

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Let A be a C^*-algebra filtered by an increasing sequence of closed ideals $\{A_n\}$ with $\overline{\cup_n A_n}=A$. Then there is a spectral sequence which converges to $K_*(A)$ and has $E^1_{p,*}=K_*(A_p/A_{p-1})$. More generally, such spectral sequences obtain for the Brown-Douglas-Fillmore functors $\mathscr{E}xt_*(A)$, the Pimsner-Popa-Voiculescu functors $\mathscr{E}xt_*(Y;A)$, the Kasparov functors $\mathscr{E}xt_*(A,B)$, or indeed for any sequence of covariant or contravariant functors on C^*-algebras which satisfies an exactness axiom.

In recent years various functors on C^*-algebras have been created by analysts interested in diverse problems in single operator theory, index theory, and classification of certain types of extensions of C^*-algebras. Notable among these is the functor $\mathscr{E}xt(A)$ of Brown-Douglas-Fillmore [1], the Pimsner-Popa-Voiculescu functor $\mathscr{E}xt(Y;A)$ [13] and the Kasparov functor $\mathscr{E}xt(A,B)$ [9]. In addition there is the variant of K-theory for Banach categories developed by Karoubi and others in great generality which yields functors K_0, K_1 on C^*-algebras (cf. [20]).

In certain cases these functors have been identified with well-known objects. For example, if X is a compact space then

$$K_q(C(X)) \cong K^q(X)$$

where $C(X)$ is the C^*-algebra of complex-valued continuous functions on X and K^q is topological K-theory which, for X compact, corresponds to formal differences of vector bundles over X. More recently, the groups $\mathscr{E}xt(C(X))$ have been computed in terms of topological K-theory [1], [7] for X compact metric of finite dimension, the groups $\mathscr{E}xt(Y;C(X))$ are known [18], [19], and the groups $\mathscr{E}xt(A,B)$ are partially understood in some special cases [9], [16].

The K_0-group of an AF-algebra has the structure of an ordered "dimension group". These groups have been characterized by Effros, Handelman, and Shen, following earlier work of Bratteli and G. Elliott. (See Effros [6] for a survey.) The groups $K_*(A\times_\alpha G)$ are known in terms of K_*A for $G=\boldsymbol{Z}$ and $\boldsymbol{R}$ by the deep work of Pimsner-Voiculescu [14] and Connes [2] respectively. These results are strong enough to determine the K-groups for many C^*-algebras of interest in applications (c.f. the striking results of Cuntz [3]).

Seen in perspective, however, the field would seem to be in its