PACIFIC JOURNAL OF MATHEMATICS
Vol. 96, No. 1, 1981

ON KILLING-RICCI FORMS OF LIE TRIPLE ALGEBRAS

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The notion of Killing-Ricci forms of Lie triple algebras is introduced as a generalization of both of Killing forms of Lie algebras and the Ricci forms of the tangent Lie triple systems of Riemannian symmetric spaces. For a class of Lie triple algebras $\mathfrak{G}$, it is shown that $\mathfrak{G}$ is decomposed into a direct sum of simple ideals if its Killing-Ricci form is nondegenerate. As an application, structure of the reductive pair consisting of a semi-simple Lie algebra and its semi-simple subalgebra is investigated.

Introduction. The concept of Lie triple algebras has been introduced, originally, by K. Yamaguti [11] as general Lie triple systems, related with locally reductive spaces of K. Nomizu [6], and treated by himself (e.g., [11]-[14]), A. A. Sagle (e.g., [8], [9]) and others. In the articles [2] and [3], the author considered Lie triple algebras as tangent algebras of homogeneous Lie loops or analytic homogeneous systems on manifolds. For the study of such algebraic systems on manifolds it seems to be very important to investigate the structure of real Lie triple algebras of finite dimension, as an extended analogy of the theory of Lie groups and Lie algebras. In this paper we consider the Killing-Ricci form β of a Lie triple algebra $\mathfrak{G}$, a symmetric bilinear form on $\mathfrak{G}$ obtained by restricting the Killing form of the standard enveloping Lie algebra of $\mathfrak{G}$. Then, under an assumption by which β becomes an invariant bilinear form on $\mathfrak{G}$, it is shown that a Lie triple algebra $\mathfrak{G}$ is decomposed into a direct sum of simple Lie triple algebra ideals, if β is nondegenerate (Theorem 2). This result is applied for a reductive pair of semi-simple Lie algebra $\mathfrak{L}$ and semi-simple subalgebra $\mathfrak{K}$ of $\mathfrak{L}$, treated by A. A. Sagle [8], [9]. Then, a direct sum decomposition of $\mathfrak{L}$ into simple Lie triple algebras and semi-simple Lie algebra ideals of $\mathfrak{K}$ is obtained (Theorem 3).

1. Preliminaries. A *Lie triple algebra* $\mathfrak{G}$ over a field F is an anti-commutative algebra over F whose multiplication is denoted by XY for $X, Y \in \mathfrak{G}$, with a trilinear operation $\mathfrak{G} \times \mathfrak{G} \times \mathfrak{G} \to \mathfrak{G}$ denoted by $D(X, Y)Z$ satisfying the following conditions for $X, Y, Z, W \in \mathfrak{G}$:

(i) $D(X, X)Z = 0$,

(ii) $\mathfrak{S}\{(XY)Z + D(X, Y)Z\} = 0$,

(iii) $\mathfrak{S}D(XY, Z)W = 0$,