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INVARIANT SUBSPACES OF NON-SELFADJOINT CROSSED PRODUCTS

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Let $\mathfrak{L}$ be the von Neumann algebra crossed product determined by a maximal abelian selfadjoint algebra $L^\infty(X)$ and an automorphism of $L^\infty(X)$. The algebra $\mathfrak{L}$ is generated by a bilateral shift L_δ and an abelian algebra $\mathfrak{M}_L$ isomorphic to L^∞; the non-selfadjoint subalgebra $\mathfrak{L}_+$ of $\mathfrak{L}$ is defined to be the weakly closed algebra generated by L_δ and $\mathfrak{M}_L$. The commutant of $\mathfrak{L}$ is the algebra $\mathfrak{R}$, also a crossed product. The invariant subspace structure of $\mathfrak{L}_+$ is investigated. It is shown that full, pure invariant subspaces for $\mathfrak{L}_+$ are unitarily equivalent by a unitary operator in $\mathfrak{R}$ if and only if their associated projections are equivalent in $\mathfrak{M}_L'$. Furthermore, a multiplicity function can be associated with each invariant subspace. The algebra $\mathfrak{R}$ contains a subalgebra $\mathfrak{R}_+$ analogous to $\mathfrak{L}_+$. It is shown that subspaces invariant for both algebras $\mathfrak{L}_+$ and $\mathfrak{R}_+$ can be parameterized in terms of certain subsets of the cartesian product $\mathbf{Z}\times X$.

1. Introduction. In their fundamental paper of 1936, Murray and von Neumann used the algebraic idea of a crossed product to exhibit special types of von Neumann algebras. The result of this construction is a crossed product of an abelian von Neumann algebra with a group of automorphisms of the algebra. This crossed product is now commonly called a group-measure algebra. Today, the study of crossed products in operator theory has become important not only for the examples it yields but also for its contribution to the general structure theory of operator algebras. For example, they have been useful in unraveling the structure of type III von Neumann algebras [11]. Indeed, Feldman and Moore [4] have recently shown that it is likely that every von Neumann algebra can be realized as a crossed product—perhaps of a complicated nature.

In this paper we shall consider crossed products of the type first considered by Murray and von Neumann [7]. We shall concentrate our attention on certain non-selfadjoint subalgebras of their crossed products, subalgebras which we call *non-selfadjoint crossed products*. These subalgebras stand in the same relation to group-measure algebras as H^∞ (the space of boundary values of bounded analytic functions on the unit disc) stands in relation to L^∞ of the circle. Two specific representations of the non-selfadjoint crossed products will be useful; these are defined in § 2 and are