PACIFIC JOURNAL OF MATHEMATICS
Vol. 97, No. 1, 1981

MANIFOLDS ADMITTING TAUT HYPERSPHERES

JAMES J. HEBDA

The definition of taut submanifolds in Euclidean space is extended to submanifolds of an arbitrary complete Riemannian manifold. Manifolds containing a tautly embedded hypersphere are characterized up to homeomorphism. Also, a partial result in this direction is proved for manifolds containing a tautly embedded sphere of arbitrary codimension.

1. Taut submanifolds have received much attention in recent years [1], [3], [6], [7]. There the emphasis is on characterizing the taut submanifolds of a particular ambient space, usually Euclidean space, although there are studies involving hyperbolic space and complex projective space as well [4], [5]. In this paper the subject is approached from a different perspective: to characterize the ambient space given that it contains certain taut submanifolds. For example:

THEOREM 1. *A complete simply connected Riemannian manifold of dimension n that admits a taut embedding of S^{n-1} is either homeomorphic to S^n, diffeomorphic to $\boldsymbol{R}^n$, or diffeomorphic to $S^{n-1} \times \boldsymbol{R}$.*

In a Euclidean sphere or in a complete, simply-connected Riemannian manifold without conjugate points, every geodesic sphere is taut. The converse is also true.

THEOREM 2. *Suppose a complete Riemannian manifold has the property that about every point some small geodesic sphere is taut. Then the manifold is either simply connected without conjugate points or isometric to a Euclidean sphere.*

2. Let M be a complete Riemannian manifold and $N \subset M$ a proper submanifold. In particular, N is a closed submanifold with the subspace topology. For each $p \in M$, we define the function $L_p: N \to \boldsymbol{R}$ by $L_p(x) = [d(x, p)]^2$ where $x \in N$ and d is the distance function on M. We say N is *taut* if for almost every $p \in M$ and almost every $r > 0$ the homeomorphism

$$i_*: H_*(L_p^{-1}([0, r])) \longrightarrow H_*(N)$$

induced by inclusion is injective, where the homology coefficients are in some field. Because of Lemma 2.8 on page 705 of [3], this definition coincides with the definition for taut submanifolds of