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COMPATIBLE PEIRCE DECOMPOSITIONS OF JORDAN TRIPLE SYSTEMS

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Jordan triple systems and pairs do not in general possess unit elements, so that certain standard Jordan algebra methods for studying derivations, extensions, and bimodules do not carry over to triples. Unit elements usually arise as a maximal sum of orthogonal idempotents. In Jordan triple systems such orthogonal sums of tripotents are not enough: in order to "cover" the space one must allow families of tripotents which are orthogonal or collinear. We show that well behaved triples and pairs do possess covering systems of mixed tripotents, and that for many purposes such nonorthogonal families serve as an effective substitute for a unit element. In particular, they can be used to reduce the cohomology of a direct sum to the cohomology of the summands.

Throughout we consider Jordan triple systems J over an arbitrary ring Φ of scalars, having product $P(x)y$ quadratic in x and linear in y with polarized trilinear product $\{xyz\} = P(x, z)y = L(x, y)z$. For easy reference we record the following standard identities satisfied by the multiplications in a Jordan triple system:

$$P(P(x)y) = P(x)P(y)P(x) \tag{0.1}$$

$$P(x)L(y, x) = P(P(x)y, x) = L(x, y)P(x) \tag{0.2}$$

$$L(P(x)y, y) = L(x, P(y)x) \tag{0.3}$$

$$L(x, y)P(z) + P(z)L(y, x) = P(\{xyz\}, z) \tag{0.4}$$

$$[L(x, y), L(z, w)] = L(\{xyz\}, w) - L(z, \{yxw\}) \tag{0.5}$$

$$\begin{aligned} P(x, y)P(z) &= L(x, z)L(y, z) - L(x, P(z)y)\ , \\ P(z)P(x, y) &= L(z, x)L(z, y) - L(P(z)x, y) \end{aligned} \tag{0.6}$$

$$\begin{aligned} P(P(x)y, z) &= P(x, z)L(y, x) - L(z, y)P(x) \\ &= L(x, y)P(x, z) - P(x)L(z, y) \end{aligned} \tag{0.7}$$

$$\begin{aligned} P(\{xyz\}) + P(P(x)P(y)z, z) &= P(x)P(y)P(z) + P(z)P(y)P(x) \\ &\quad + L(x, y)P(z)L(y, x) \end{aligned} \tag{0.8}$$

(see [2], [3], [8] for basic facts about Jordan triple systems).

We recall the 3 basic examples of Jordan triple systems. The *rectangular* $p \times q$ *matrices* with entries in a unital algebra D with