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ESTIMATES OF MEROMORPHIC FUNCTIONS AND SUMMABILITY THEOREMS

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The main goal of this paper is to prove the following theorem.

THEOREM 1. **Let L be an unbounded operator in a Hilbert space $\mathfrak{H}$, having a discrete spectrum $\{\lambda_j\}\subset G = B_R \cup P_{q,h}$, where $B_R = \{\lambda: |\lambda| \leqq R\}$, $P_{q,h} = \{\lambda: \operatorname{Re}\lambda \geqq 0,\ |\lambda|>1,\ |\operatorname{Im}\lambda| \leqq h(\operatorname{Re}\lambda)^q,\ h>0,\ -\infty<q<1\}$, and for some $\gamma<\infty$, $L^{-1}\in\sigma_\gamma$. Also let the estimate**

$$\|(I\lambda - L)^{-1}\| \leqq Cd^{-1}(\lambda, G),\ \lambda \bar{\in} G$$

hold outside the domain $G' = B_R \cup P_{q,2h}$, and for some $a>0$, $p>0$

$$\sum_{|\lambda_j|\leqq t} 1 = n(t) \leqq dt^p$$

provided t is sufficiently large.

Then $L \in A(\alpha, \mathfrak{H})$ for any $\alpha > \max 0,\ p-(1-q)$.

Besides, if the numbers a or h can be chosen arbitrarily small and $p-(1-q)>0$, then $\alpha = p-(1-q)$ is admissible.

Introduction. Let L be an unbounded linear operator in a separable complex Hilbert space $\mathfrak{H}$ with domain of definition $\mathscr{D}(L)$ which is dense in $\mathfrak{H}$, having a discrete spectrum $\sigma(L)$. Let $\{e_j\}_{j=1}^{\infty}$ be a sequence consisting of bases in the root subspaces of L, where e_j is a root vector corresponding to the eigenvalue λ_j. To each vector $x\in\mathfrak{H}$ we associate its Fourier series $\sum(x, e_j^*)e_j$ with respect to this system (not necessarily convergent), where $\{e_j^*\}$ is a system which is biorthogonal to $\{e_j\}$.

We write $L\in\mathscr{A}(\alpha, \mathfrak{M}, \mathfrak{H})$ if for an arbitrary vector x in $\mathfrak{M}$, where $\mathfrak{M}$ is some linear manifold in $\mathfrak{H}$, the Fourier series $\sum(x, e_j^*)e_j$ is summable in $\mathfrak{H}$ to x by the Abel method of order α with parenthesis.

If we suppose that L has no associated vectors and all its eigenvalues $\{\lambda_j\}$ lie in the sector $\Lambda_\theta = \{\lambda: |\arg\lambda| \leqq \pi/2\theta,\ 1/2 \leqq \theta < \infty\}$ then the Abel method of summability of order $\alpha(\alpha\leqq\theta)$ consists in replacing the series $\sum(x, e_j^*)e_j$ by series

$$u_x(t) = \sum_{j=1}^{\infty} e^{-\lambda_j^{\alpha}t}(x, e_j^*)e_j\,; \tag{1}$$

it is required that for any $t>0$ after possible recombination of its terms and appropriate use of parenthesis (not depending on $x\in\mathfrak{M}$, or $t>0$) this series converges in $\mathfrak{H}$ and its sum $u_x(t)$ converges