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A GENERAL VERSION OF VAN DER CORPUT'S DIFFERENCE THEOREM

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Let $\omega(n)$ be a real-valued sequence, and let us assume that for all positive integers g the difference-sequences $\Delta_g\omega(n) = \omega(n+g) - \omega(n)$ are uniformly distributed modulo 1, then $\omega(n)$ itself is uniformly distributed modulo 1. This is van der Corput's difference theorem or the so-called main theorem in the theory of uniform distribution. In this paper I present a rather generalized version of this theorem which not only enables us to prove the original van der Corput theorem, but also the general approximation theorem of Kronecker in its discrete and in its continuous version. Moreover, a few other examples of uniformly distributed sequences can be constructed by this general difference theorem.

The proof of the difference theorem was given by van der Corput in using his so-called "fundamental inequality". M. Tsuji, E. Hlawka, J. H. B. Kemperman and R. J. Taschner formulated stronger versions of this fundamental inequality and by doing this reached to new aspects of the difference theorem. A different approach to the difference theorem was investigated by J. Bass and J.-P. Bertrandias: they related it to the Bochner-Herglotz representation theorem about positive definite functions. This method was further elaborated by J. Cigler. In particular, Cigler proved the uniform distribution of $n\alpha$ modulo 1 for irrational α only by using his version of the difference theorem. Cigler tried to extend his theorem for uniformly distributed functions, defined on a local compact commutative group. He recognized, however, that the topology of the group troubles the application of the difference theorem. It is, for example, impossible to derive Hlawka's difference theorem about C-uniformly distributed functions by Cigler's method.

In this paper we will follow the way, indicated by Bass, Bertrandias, and Cigler, but we separate the domain of the uniformly distributed functions from the group which makes it possible to define difference sequences. By doing this, the difficulties raised by the topology of the group can be avoided.

Before formulating the main theorem, we list all necessary prerequisites: Let X be a compact group and let χ be the normed Haar measure on X. E^h, $h \in H$, designates a "main system" of representations of X, this means: a set of irreducible unitary representations so that each irreducible