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LIMIT CIRCLE TYPE RESULTS FOR SUBLINEAR EQUATIONS

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Recently there has been an interest in obtaining integrability criteria for solutions of nonlinear differential equations similar in nature to those known for linear equations. In the classic paper on the subject, H. Weyl classified the second order linear differential equation

$$(a(t)x')' + q(t)x = 0 \tag{1}$$

as being of the limit circle type if all its solutions are square integrable, i.e.,

$$\int^{\infty} x^2(u)\,du < \infty;$$

otherwise the equation was said to be of the limit point type. In this paper we discuss extensions of the limit point-limit circle classification to forced second order nonlinear equations of the type

$$(a(t)x')' + q(t)f(x) = r(t). \tag{2}$$

Throughout this paper we will assume that $a, q, r\colon [t_0, \infty) \to R$ and $f\colon R \to R$ are continuous, $a', q' \in AC_{\mathrm{loc}}[t_0, \infty)$, $a'', q'' \in L^2_{\mathrm{loc}}[t_0, \infty)$, $a(t) > 0$, $q(t) > 0$ and $xf(x) \geq 0$ for all x. We will say that equation (2) is of nonlinear limit circle type if every solution $x(t)$ of (2) satisfies

$$\int_{t_0}^{\infty} x(u)f(x(u))\,du < \infty,$$

and we will say that equation (2) is of nonlinear limit point type otherwise. (For a discussion of other possible definitions of nonlinear limit point and limit circle we refer the reader to the papers of Atkinson [**1**] and Graef [**7**].) This of course reduces to the square integrability of solutions in the case of equation (1). While some authors have discussed the nonlinear limit point-limit circle problem (see [**1**, **3**, **5**, **7**, **9**, **12**, **13**, **14**, **16**]), the majority of the results obtained have been of the nonlinear limit point type for unforced equations. In fact only the papers of Graef [**7**] and Spikes [**12**, **13**] contain limit circle results for equation (2). Moreover, in the papers written on the nonlinear limit point-limit circle problem to date, the case of $f(x)$ being sublinear has either been ignored completely or explicitly excluded by hypothesis from consideration. It is our purpose here to consider this case exclusively. Henceforth we consider the equation

$$(a(t)x')' + q(t)x^{\gamma} = r(t) \tag{3}$$