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BUNDLES OVER CONFIGURATION SPACES

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Let $F(R^n, k)$ be the configuration space of ordered sets of k distinct points in R^n. $F(R^n, k)$ is acted upon freely by the symmetric group on k letters, Σ_k. In this paper we calculate the order of the vector bundles

$$\xi_{n,k}\colon F(R^n, k) \times_{\Sigma_k} R^k \to F(R^n, k)/\Sigma_k.$$

Applications to the study of iterated loop spaces of spheres are also discussed.

1. The study of the stable homotopy type of the spaces $\Omega^n S^{n+r}$ has received much attention in recent years [**2, 8, 13**]. The starting point for this study was Snaith's stable descomposition [**18**]:

$$\Omega^n S^{n+r} \simeq_s \bigvee_{k \geq 0} F(\mathbf{R}^n, k)^+ \wedge_{\Sigma_k} S^{r^{(k)}},$$

where $F(\mathbf{R}^n, k)^+$ is the configuration space of k ordered distinct points in $\mathbf{R}^n$ together with a disjoint basepoint, $S^{r^{(k)}}$ is the k-fold smash product of S^r with itself, Σ_k is the symmetric group of k letters, and where "$\simeq_s$" denotes stable homotopy equivalence.

The space $F(\mathbf{R}^n, k)^+ \wedge_{\Sigma_k} S^{r^{(k)}}$ is clearly the Thom complex of the r-fold Whitney sum of the vector bundle

$$\xi_{n,k}\colon F(\mathbf{R}^n, k) \times_{\Sigma_k} \mathbf{R}^k \to F(\mathbf{R}^n, k)/\Sigma_k.$$

If $M(\xi_{n,k})$ is the associated Thom spectrum, then Snaith's theorem gives an equivalence of spectra

$$\Sigma^\infty \Omega^n S^{n+r} \simeq \bigvee_{k \geq 0} \Sigma^{rk} M(r\xi_{n,k}),$$

where Σ^∞ is the stabilization functor which assigns to a space its associated suspension spectrum.

If $\phi_{n,k}$ is the stable order of $\xi_{n,k}$ (i.e., $\phi_{n,k}$ is the smallest integer such that $\phi_{n,k}\xi_{n,k}$ is stably trivial) then we have the obvious periodicity

$$M((r + \phi_{n,k})\xi_{n,k}) \simeq \Sigma^{k\phi_{n,k}} M(r\xi_{n,k}).$$

This, together with Snaith's theorem gives clear interrelationships amongst the stable homotopy types of the spaces $\Omega^n S^{n+r}$ as r varies.

The case $n = 2$ is well understood by the work of F. Cohen, M. Mahowald, and R. J. Milgram [**5**], who proved that $\phi_{2,k} = 2$ for all k. The resulting periodicity in the homotopy type of the associated Thom