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NOTES ON THE FEYNMAN INTEGRAL, III: THE SCHROEDINGER EQUATION

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In the setting of Cameron and Storvick's recent theory we show that the solution of an integral equation formally equivalent to the Schroedinger equation is expressible as the analytic Feynman integral of a function on ν-dimensional Wiener space of the form $F(\vec{X}) = \exp\{\int_0^t \theta(t-s, \vec{X}(s)+\vec{\xi})\,ds\}\psi(\vec{X}(t)+\vec{\xi})$. Here $\vec{X}$ is an $\mathbf{R}^\nu$-valued continuous function on $[0, t]$ such that $\vec{X}(0)=\vec{0}$, $\vec{\xi}\in\mathbf{R}^\nu$, and ψ and $\theta(s,\cdot)$ are Fourier-Stieltjes transforms.

1. Introduction. Let $L_2^\nu[0, t_0] = L_2^\nu$ denote the space of $\mathbf{R}^\nu$-valued, Lebesgue measurable, square integrable functions on $[0, t_0]$. Let $C^\nu[0, t_0]$ denote Wiener space, that is the space of $\mathbf{R}^\nu$-valued, continuous functions $\vec{X}$ on $[0, t_0]$ such that $\vec{X}(0) = \vec{0}$. In a recent paper [4], Cameron and Storvick introduced a Banach algebra S of (equivalence classes of) functions on Wiener space which are a kind of stochastic Fourier transform of Borel measures on L_2^ν. (Precise definitions will be given in §2.) For such functions they showed that the analytic Feynman integral, defined by analytic continuation of the Wiener integral, exists. Further they showed that functions of the form

$$F(\vec{X}) = \exp\left\{\int_0^{t_0}\theta(s, \vec{X}(s))\,ds\right\} \tag{1.1}$$

are in S where they assumed that the "potential" θ: $[0, t_0]\times\mathbf{R}^\nu \to \mathbf{C}$ satisfies: (i) For each s in $[0, t_0]$, $\theta(s,\cdot)$ is the Fourier-Stieltjes transform of an element σ_s of $M(\mathbf{R}^\nu)$, the space of $\mathbf{C}$-valued, countably additive (and hence bounded) Borel measures on $\mathbf{R}^\nu$; that is ($\langle\,,\rangle$ denotes inner product in $\mathbf{R}^\nu$)

$$\theta(s, \vec{U}) = \int_{\mathbf{R}^\nu}\exp\{i\langle\vec{U}, \vec{V}\rangle\}\,d\sigma_s(\vec{V}). \tag{1.2}$$

(ii) For each Borel subset E of $[0, t_0]\times\mathbf{R}^\nu$, $\sigma_s(E^{(s)})$ is a Borel measurable function of s on $[0, t_0]$. Here $E^{(s)}$ denotes the s-section of E. (iii) The total variation $\|\sigma_s\|$ of σ_s is bounded as a function of s.

For the case $\nu = 1$ and under strengthened measurability assumptions, Cameron and Storvick showed in [5] that the analytic Feynman integral of functions F which are essentially of the form (1.1) gives a solution to an integral equation formally equivalent to Schroedinger's equation. In [5] Cameron and Storvick make use of the fact that F is in S. This was