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NATURAL TRANSFORMATIONS OF TENSOR-PRODUCTS OF REPRESENTATION-FUNCTORS I, COMBINATORIAL PRELIMINARIES

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The present paper furnishes some combinatorial preliminaries towards a study of natural transformations between tensor products of shape functors $\wedge^{\alpha}$ and co-shape functors $\vee_{\alpha}$. The main result is the construction of an explicit basis for the module defined by (1) below; an apparently new result used for this purpose, which may be of some independent interest, is a 'column-free' expression for the Young idempotent NPN (in Young's terminology) associated with a partition, given by 1.2 below.

Introduction. In the following, the reader will be assumed to be familiar with the concepts and results of [**1**] and [**2**].

Let $\alpha_1,\ldots,\alpha_m,\beta_1,\ldots,\beta_n$ be partitions, and let A be a commutative ring. The present paper is the first of a series concerned with the A-module, denoted by

$$(1) \qquad \operatorname{Nat}\operatorname{Tsf}_A(\alpha_1 \times \cdots \times \alpha_m, \beta_1 \times ,\ldots, \times \beta_n),$$

which consists of all natural transformations from the functor

$$\wedge_A^{\alpha_1} \otimes \cdots \otimes \wedge_A^{\alpha_m} \colon \mathbf{Mod}_A \to \mathbf{Mod}_A,\ E \mapsto \wedge_A^{\alpha_1} E \otimes \cdots \otimes \wedge_A^{\alpha_m} E$$

into the similar functor $\wedge_A^{\beta_1} \otimes \cdots \otimes \wedge_A^{\beta_n}$ (If A is a field this is equivalent to studying the space of interwining operators between the two representations of $\mathrm{GL}(E)$ with representation-modules $\wedge_A^{\alpha_1} E \otimes \cdots \otimes \wedge_A^{\alpha_m} E$ and $\wedge_A^{\beta_1} E \otimes \cdots \otimes \wedge_A^{\beta_n} E$ respectively (provided $\dim E$ is sufficiently great).

When A is a $\mathbf{Q}$-algebra, a generating set for the A-module (1) is furnished by the "exchange-transformations" given by Def. 3–6 below and a free basis by the subset of these given by Def. 3–8 (In the case $m = 2$, $n = 1$ this furnishes a more precise version of the Littlewood-Richardson rule (which only specifies the cardinality of such a basis).) The general case does not seem to be an immediate consequence of this special case; the attempt to reduce to the special case in the obvious way, by using the associativity of the tensor product, leads to the problem next to be discussed (and yields a second, different free basis for 1), related to