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NONSMOOTH ANALYSIS ON PARTIALLY ORDERED VECTOR SPACES: PART 2–NONCONVEX CASE, CLARKE'S THEORY

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The purpose of this paper is to extend the recently developed Clarke theory of generalized gradients to vector valued mappings. For that we introduce the notion of locally o-Lipschitz mappings and develop a subdifferential calculus for them. In this process, we have the opportunity for comparison with analogous results obtained in the convex case.

1. Introduction. We know (see [**14**]) that a convex mapping majorized in a neighborhood of a point is locally o-Lipschitz in the interior of its domain. So it is natural to go a step further and ask whether we can have an analogous subdifferential theory for locally o-Lipschitz mappings.

In the real valued case, this problem was first introduced and successfully solved by Clarke [**2**]—[**7**]. After Clarke, others have also contributed in this or parallel directions, e.g., Aubin [**1**], Halkin [**9**], Hiriart-Urruty [**11**]—[**14**], Rockafellar [**22**], and Warga [**27**].

In this paper, we construct a similar theory for vector valued mappings. Having as our starting point the locally o-Lipschitz mappings, we define the generalized gradient for such mappings, and using that we develop a complete subdifferential theory. Although we face serious analytical difficulties working with vector valued mappings (lack of functional separability results), nevertheless introducing the notion of generalized o-directional derivatives and using the results obtained in [**19**], we are able to obtain several new results that will be potentially useful in solving nonsmooth, nonconvex vector valued extremal problems. Similar work was done very recently by Ioffe [**16**], [**17**] and Thibault [**15**]. In the last section of this paper, we will compare our results with those obtained in the above-mentioned works.

All through this paper, X will be a Banach space and Y an order complete Banach lattice. Any additional assumptions will be mentioned explicitly. The definitions and notational conventions are the same as those introduced in §2 of [**19**].

In the next section we introduce the locally o-Lipschitz mappings, which play an important role in this theory, and we examine several of their properties.