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A CLASS OF SURJECTIVE CONVOLUTION OPERATORS

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Let μ be a distribution with compact support in $\mathbf{R}^n$. In the terminology of Ehrenpreis [2] μ is called invertible for a space of distributions $\mathcal{F}$ in $\mathbf{R}^n$ if $\mu * \mathcal{F} = \mathcal{F}$. Using his characterisation of invertible distributions in terms of the growth of their Fourier transforms, we obtain a class of invertible distributions which properly contains the distributions with finite supports. We consider $\mathcal{F} = \mathcal{E}$ (or $\mathcal{D}'$) and $\mathcal{F} = \mathcal{D}'_F$, but our results for the latter space are only partial.

1. Introduction. We follow the notation of Schwartz [**6**]: by $\mathcal{D}'$ ($\mathcal{D}'_F$) we denote the space of distributions (distributions of finite order) in $\mathbf{R}^n$. $\mathcal{E}$ will denote the space of infinitely differentiable functions in $\mathbf{R}^n$ with the topology of uniform convergence of functions and all their derivatives on compact subsets of $\mathbf{R}^n$. The dual space of $\mathcal{E}$, denoted by $\mathcal{E}'$, consists of distributions with compact support in $\mathbf{R}^n$. For $\mu \in \mathcal{E}'$ we define the Fourier-Laplace transform of μ by

$$\hat{\mu}(\zeta) = \mu(e^{-i\langle \cdot, \zeta \rangle}), \qquad \zeta \in \mathbf{C}^n.$$

Ehrenpreis [**2**] and Hörmander [**3**] have studied the range of convolution operators

$$u \mapsto \mu * u, \qquad \mu \in \mathcal{E}', \tag{1}$$

in each of the spaces $\mathcal{D}'$, $\mathcal{D}'_F$ and $\mathcal{E}$. We recall their main result: the operator (1) in $\mathcal{E}$ and, equivalently, in $\mathcal{D}'$ (resp. in $\mathcal{D}'_F$) is surjective if and only if $\hat{\mu}$ is slowly decreasing (resp. very slowly decreasing) in the sense of

DEFINITION 1. Let $\mu \in \mathcal{E}'$. $\hat{\mu}$ is called slowly decreasing if there exist constants A, B and m such that

$$\sup_{|\xi - \xi_0| \le A \log(2 + |\xi_0|)} |\hat{\mu}(\xi)| \ge B(1 + |\xi_0|)^{-m}$$

for all $\xi_0 \in \mathbf{R}^n$. $\hat{\mu}$ is called very slowly decreasing if there exists a constant m and for each $\varepsilon > 0$ a constant B_ε such that

$$\sup_{|\xi - \xi_0| \le \varepsilon \log(2 + |\xi_0|)} |\hat{\mu}(\xi)| \ge B_\varepsilon(1 + |\xi_0|)^{-m}$$

for all $\xi_0 \in \mathbf{R}^n$.