PACIFIC JOURNAL OF MATHEMATICS
Vol. 110, No. 2, 1984

ON THE G-COMPACTIFICATION OF PRODUCTS

JAN DE VRIES

Let $\beta_G X$ denote the maximal equivariant compactification (G-compactification) of the G-space X (i.e. a topological space X, completely regular and Hausdorff, on which the topological group G acts as a continuous transformation group). If G is locally compact and locally connected, then we show that $\beta_G(X \times Y) = \beta_G X \times \beta_G Y$ if and only if $X \times Y$ is what we call G-pseudocompact, provided X and Y satisfy a certain non-triviality condition. This result generalizes Glicksberg's well-known result about Stone-Čech compactifications of products to the case of topological transformation groups.

1. Introduction. In this paper we prove a generalization to the case of topological transformation groups of Glicksberg's well-known result about Stone-Čech compactifications of products. Recall, that a topological space X is *pseudocompact*, whenever $C(X) = C^*(X)$, i.e. every continuous real-valued function on X is bounded. A convenient characterization of pseudocompactness of a completely regular Hausdorff space X is that X contains no infinite sequence of non-empty open subsets which is locally finite. Cf. **[4]** and, for more about pseudocompactness, **[5]**. Glicksberg's theorem says that if X and Y are infinite completely regular spaces, then $\beta(X \times Y) = \beta X \times \beta Y$ if and only if $X \times Y$ is pseudocompact. See **[6]** and also **[4]** and **[10]** for short proofs. Adopting the techniques of **[4]** and **[10]**, we were able to prove (terminology will be explained in 1.1 and 2.1 below):

THEOREM. *Let G be a locally compact, locally connected topological Hausdorff group, and let X and Y be two G-infinite, completely regular Hausdorff G-spaces. Then $\beta_G(X \times Y) = \beta_G X \times \beta_G Y$ if and only if $X \times Y$ is G-psuedocompact.*

Before explaining the terminology we wish to point out two shortcomings of our result. First, we did not yet succeed in reducing the case of infinite products to the case of finite products (cf. **[10]**). The second remark concerns the condition that X and Y have to be what we call G-infinite. It is clear why Glicksberg's theorem has to contain the condition that X and Y are infinite: if either X or Y is finite, then always $\beta(X \times Y) = \beta X \times \beta Y$ without any further condition on $X \times Y$. However, compared with this situation, our "non-triviality condition" in the