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JORDAN TRIPLE SYSTEMS WITH COMPLETELY REDUCIBLE DERIVATION OR STRUCTURE ALGEBRAS

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We prove that a finite-dimensional Jordan triple system over a field k of characteristic zero has a completely reducible structure algebra iff it is a direct sum of a trivial and a semisimple ideal. This theorem depends on a classification of Jordan triple systems with completely reducible derivation algebra in the case where k is algebraically closed. As another application we characterize real Jordan triple systems with compact automorphism group.

The main topic of this paper is finite-dimensional Jordan triple systems over a field of characteristic zero which have a completely reducible derivation algebra.

The history of the subject begins with [7] where G. Hochschild proved, among other results, that for an associative algebra $\mathcal{A}$ the derivation algebra is semisimple iff $\mathcal{A}$ itself is semisimple. Later on R. D. Schafer considered in [**18**] the case of a Jordan algebra $\mathcal{J}$. His result was that Der $\mathcal{J}$ is semisimple if and only if $\mathcal{J}$ is semisimple with each simple component of dimension not equal to 3 over its center. This theorem was extended by K.-H. Helwig, who proved in [**6**]:

Let $\mathcal{J}$ be a Jordan algebra which is finite-dimensional over a field of characteristic zero. Then the following are equivlent:

(1) Der $\mathcal{J}$ is completely reducible and every derivation of $\mathcal{J}$ has trace zero,

(2) $\mathcal{J}$ is semisimple,

(3) the bilinear form on Der $\mathcal{J}$ given by $(D_1, D_2) \to \operatorname{trace}(D_1 D_2)$ is non-degenerate and every derivation of $\mathcal{J}$ is inner.

After some preparations in §§1—3 we will show in §4 that the same theorem holds for Jordan triple systems. The proof in this case is different from the Jordan algebra case. It relies on a classification of Jordan triple systems whose derivation algebras are completely reducible. It is easy to see that V is an example for such a triple system, if

(a) V is semisimple or if

(b) V is trivial, i.e. all products vanish.