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RIESZ SETS AND A THEOREM OF BOCHNER

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Conditions are developed from which one may infer that a measurable subset S of an LCA group $\hat{G}$ is a Riesz set, or in general, a small p set. Subsets of inverse images of small p sets under continuous homomorphisms which are small q sets for some q are investigated. These considerations yield extensions to LCA groups of Bochner's version of the F. and M. Riesz theorem.

The aim of this article is to develop criteria which, under appropriate circumstances, imply that a subset of an LCA group is a Riesz set, or in general, a small p set. The conditions developed lead one rather naturally to consider the behavior of inverse images of Riesz sets under continuous homomorphisms, specifically to consider if or when the inverse image of a Riesz set, or a subset thereof, is again a Riesz set. Special emphasis is given to this aspect of the problem, and, in those cases where a solution is obtained, to certain applications which arise as a consequence.

The notion of a Riesz set has its origins in the F. and M. Riesz theorem, which provides the prototype for all Riesz sets, namely, the nonnegative integers Z^+. This renowned theorem states that if the Fourier transform $\hat{\mu}$ of a measure on $[0, 2\pi)$ vanishes off Z^+, then μ is absolutely continuous with respect to Lebesgue measure ([**9**], [**10**, 8.2.1]). By analogy, if G is an LCA group and S is a measurable subset of $\hat{G}$, then one calls S a Riesz set if each measure $\mu \in M(G)$ whose transform $\hat{\mu}$ vanishes off S is absolutely continuous with respect to Haar measure on G. Any measurable subset of $\hat{G}$ having finite Haar measure in $\hat{G}$ is a Riesz set, this being a consequence of the Inversion theorem ([**5**, 31.33]). Bochner's well-known generalization of the F. and M. Riesz theorem to Z^n provides a multitude of nontrivial Riesz sets in Z^n ([**1**, Theorem 5], [**10**, 8.2.5]). Loosely speaking, for $n = 2$ this theorem states that the set of points in any "angle" in $Z \times Z$ of less than π radians is a Riesz set. In particular, the Cartesian product $Z^+ \times Z^+$ of the Riesz set Z^+ is again a Riesz set. As we shall see later, versions of Bochner's theorem continue to hold in the general group setting ("Bochner's theorem" in the sequel will refer to the theorem mentioned above, as opposed to the one having to do with positive-definite functions).