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WEAK FACTORIZATION OF DISTRIBUTIONS IN H^p SPACES

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The weak factorization theorem for real Hardy spaces $H^p(\mathbf{R}^n)$, previously obtained by Coifman, Rochberg and Weiss, and by Uchiyama for the case $p > n/(n+1)$, is extended to the case $p \leq n/(n+1)$.

1. Introduction. The purpose of this paper is to give an extension of the following

THEOREM A. (*Coifman-Rochberg-Weiss* [**3**; *Theorem* II], *Uchiyama* [**7**; *Corollary to Theorem* 1], [**8**].) *Let K be a homogeneous singular integral operator of Calderón-Zygmund type on $\mathbf{R}^n$ and K' its conjugate. Suppose $p, q, r > 0$ satisfy $1/p = 1/q + 1/r < 1 + 1/n$.* (i) *If $h \in L^2 \cap H^q(\mathbf{R}^n)$, $g \in L^2 \cap H^r(\mathbf{R}^n)$ and*

$$f = hKg - gK'h,$$

then $f \in H^p(\mathbf{R}^n)$ and

$$\|f\|_{H^p} \leq C_1 \|h\|_{H^q} \|g\|_{H^r}.$$

(ii) *Conversely, if, furthermore, K is not a constant multiple of the identity operator and $p \leq 1$, every $f \in H^p(\mathbf{R}^n)$ can be written as*

$$f = \sum_{j=1}^{\infty} \lambda_j \left(h_j K g_j - g_j K' h_j \right),$$

where λ_j are complex numbers, $h_j \in L^2 \cap H^q(\mathbf{R}^n)$, $g_j \in L^2 \cap H^r(\mathbf{R}^n)$ and

$$\|h_j\|_{H^q} \|g_j\|_{H^r} \leq C_2, \qquad \left(\sum_{j=1}^{\infty} |\lambda_j|^p \right)^{1/p} \leq C_3.$$

The constants C_1, C_2 and C_3 depend only on p, q, r, K and n.

As for the definition of $H^p(\mathbf{R}^n)$, see Fefferman-Stein [**4**]; as for the operators K and K', see the definitions given in the next section.

An extension of part (i) to the case $1/p \geq 1 + 1/n$ is given in the following

THEOREM B. (*Miyachi* [**6**].) *Let $K_1, \ldots, K_N$ be homogeneous singular integral operators of Calderón-Zygmund type on $\mathbf{R}^n$ and K_j' their conjugates.*