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# REPRESENTATION OF VECTOR-VALUED FUNCTIONS BY LAPLACE TRANSFORMS

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**In this note various criteria are given which solve the following problem. Given a locally convex space $X$ and an $X$-valued function on $(0, \infty)$, when does there exist an $X$-valued function on $[0, \infty)$, usually required to have certain specific properties such as continuity, integrability, etc., whose Laplace transform is the given function? Some of these criteria are new even in Banach spaces.**

**1. Introduction.** Many problems in classical analysis are subsumed under the theory of semigroups of linear operators. This is particularly true of Cauchy's problem which arises in the theory of partial differential equations. An important problem is the generation of semigroups, that is, to determine which operators are the infinitesimal generator of a semigroup. For strongly continuous semigroups a successful approach to this problem can be based on the Post-Widder inversion formula for Laplace transforms, which provides a connection between the semigroup and the resolvent map of its infinitesimal generator [**12**]. However, as W. Feller repeatedly emphasized (see for example [**7**]–[**10**]), the theory of strongly continuous semigroups is often inadequate in practice. For example, it is not applicable to the theory of diffusion processes and problems arising from applications to stochastic processes. The semigroup arising from the heat equation may also fail to be strongly continuous in certain function spaces; see §3 of [**2**]. It seems plausible that a theory for semigroups which are not necessarily strongly continuous could still be based on the method of Laplace transforms. However, the representation theorems available for Laplace transforms of vector-valued functions appear to be too restrictive for such applications.

Accordingly, the aim of this note is to provide more general criteria which guarantee that a given vector-valued function is a Laplace transform. The results are based on the classical Widder inversion operators (see §2). The functions involved will assume their values in locally convex spaces. This is not because they are more general than Banach spaces, but because they provide the natural setting for problems of this type.

A novelty of the note occurs in §5 where sufficient conditions are presented which ensure that a given function is the Laplace transform of a