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ON THE HOMOLOGY OF SPACES OF SECTIONS OF COMPLEX PROJECTIVE BUNDLES

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By means of a Moore-Postnikov decomposition we compute the first homology groups of some spaces of sections of projective bundles associated to complex vector bundles.

1. Introduction. Let $P\xi$: $P(V) \to X$ be the projective bundle associated to a complex $(n+1)$-dimensional vector bundle ξ: $V \to X$, $n \geq 1$, over a connected CW-complex X. Suppose that $P\xi$ admits a section u: $X \to P(V)$ and consider the space Γ_u of all sections vertically homotopic to u. In this paper we discuss the (co)-homology of Γ_u using the construction by Thom-Haefliger [1] of Γ_u as an inverse limit derived from the Moore-Postnikov factorization of $P\xi$. Explicit formulas for some (co-)homology groups of Γ_u are obtained provided $X = T$ is a closed, orientable surface, $X = P^m$, $1 \leq m \leq n$, is a complex projective space, or $X = L^{2m+1}(p)$, $1 \leq m < n$, p odd, is a lens space.

If ξ is trivial, then Γ_u is a (path-)component of the space $M(X, P^n)$ of maps of X into P^n, so in particular we obtain formulas for some homology groups of the components of $M(X, P^n)$. In fact, sufficient information is obtained to show that two components of $M(T, P^n)$ or $M(P^m, P^n)$, $1 \leq m \leq n$, are homotopy equivalent if and only if their associated degrees have the same absolute value.

The work presented here was inspired by the paper [4], in turn inspired by [2], in which Larmore and Thomas computed the fundamental group of some spaces of sections of real projective bundles associated to real vector bundles. In contrast to [4] we avoid, however, the use of twisted coefficients, for $P\xi$ is orientable, and the focus will be on homology groups rather than homotopy groups.

2. Moore-Postnikov factorizations of projective bundles. Since the projective bundle $P\xi$: $P(V) \to X$, having a connected structure group, is