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AN ISOPERIMETRIC INEQUALITY FOR SURFACES STATIONARY WITH RESPECT TO AN ELLIPTIC INTEGRAND AND WITH AT MOST THREE BOUNDARY COMPONENTS

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Let M be a connected C^2 two dimensional submanifold with boundary of $\mathbf{R}^3$, with at most three boundary components. Let Φ be a positive even elliptic parametric integrand of degree two on $\mathbf{R}^3$ ([5]), and suppose that M is stationary with respect to Φ. In this paper we show that there is a constant $C(\Phi)$ such that M satisfies the isoperimetric inequality

$$L^2 \geq C(\Phi)A, \tag{1.1}$$

where L is the length of ∂M and A is the surface area of M. In the proof we also prove a lemma that M satisfies the inequality

$$\text{length}(\partial M) \geq C(\Phi) \text{ diameter } M. \tag{1.2}$$

In the case that M is simply connected (1.1) follows for $C(\Phi) = 4\pi$ from the fact that such a surface must have nonpositive Gauss curvature [4]. In the case that ∂M has two components and Φ is the parametric area integrand the inequality (1.1) with $C = 4\pi$ has been proven by Osserman and Schiffer, [9]. More generally, an inequality of the form (1.1) has been proven for area stationary k dimensional varifolds on $\mathbf{R}^n$ by Allard, [2]. For the case that M has two or three boundary components and Φ is different from the area integrand the results (1.1), (1.2) are new. We note that this result also allows us to obtain lower bounds on area for such a manifold M using (1.1) together with the techniques of [1], [9]. For a review of other results on the isoperimetric inequality see the paper by Osserman [7].

In many isoperimetric inequality proofs, the equation

$$2A = -2\int_M (x - c) \cdot H + \int_{\partial M} (x - c) \cdot \nu \tag{1.3}$$

plays a central role, where $c \in \mathbf{R}^3$, H is the mean curvature vector of M, and ν is the exterior normal of ∂M with respect to M. For example, see Osserman [7], pp. 1203–1204. In the present work a similar equation is used where H is replaced by a weighted combination of the principal