LIFTINGS OF SUPERCUSPIDAL REPRESENTATIONS
OF Gl$_2$

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Let F be a p-field. Let E/F be a tamely ramified cyclic extension of odd degree. Denote by $\pi_{E/F}$ and Π respectively, the lift and the Shintani lift of an irreducible supercuspidal representation π of $\mathrm{Gl}_2(F)$. The comparison of these two lifts of π is made by breaking up the formula for the character of a supercuspidal representation into a sum over a certain set of double cosets. As a result, we show that the liftings $\pi_{E/F}$ and Π are equivalent.

Let F be a p-field; that is, the completion under the p-adic topology of either an algebraic number field or an algebraic function field. Let W_F be the absolute Weil group of F. Then it is a conjecture of Langlands that there should exist a "natural" map $\sigma \mapsto \pi(\sigma)$ between the set $A_d(F)$ of the continuous d-dimensional representations of W_F and a certain subset of the set $A(\mathrm{Gl}_d(F))$ of admissible irreducible representations of the general linear group $\mathrm{Gl}_d(F)$. (For the history and current status of this problems see for example [J-L], [Sh], [K4]; for generalizations see [B]).

Since the map $\sigma \mapsto \pi(\sigma)$ should be natural, we may expect, among other things, that the map which sends a d-dimensional representation σ of W_F to its restriction σ_E of W_E should correspond to a map which sends irreducible admissible representations of $\mathrm{Gl}_d(F)$ to irreducible admissible representations of $\mathrm{Gl}_d(E)$. Two candidates for this latter map have been proposed in different contexts by Shintani and Kutzko when $d = 2$ (see [L], [K4]). Shintani's map comes about from global considerations and is defined in fact as a map on characters in the case that the extension E/F is cyclic of prime degree. Kutzko's map is defined in terms of the representations and plays a central role in his proof of the correspondence in the case of $d = 2$ (see [K4]). However, it is defined only in case the extension is tamely ramified. In order to better understand the nature of Langland's proposed correspondence in dimensions greater than two it is thus of importance to compare these two maps. This will be our goal in what follows.

In §1 the requisite definitions and preliminaries are provided to describe the representation theory of Gl_2. In particular, the set of supercuspidal representations (those representations which should correspond